CHAPTER 4
FLUX, SPECIFIC INTENSITY AND OTHER ASTROPHYSICAL TERMS

4.1 *Introduction*

In previous chapters we have used the definitions and symbols of the quantities in radiation theory as recommended by various bodies. We mentioned that, in the context of the special needs of stellar atmosphere theory, usage sometimes differs from the standard. In this chapter we introduce the definitions and symbols that are commonly used in stellar atmosphere theory, and we continue to use the astronomical usage henceforth.

In a later paragraph you will be asked to imagine a "horizontal" surface in the atmosphere of a star. Lest you are thinking of a star as a large, ball-shaped thing, and the word "horizontal" is puzzling, let me say that, at least for the time being, I am considering a "shallow" atmosphere; that is, an atmosphere whose depth is very small compared with the radius of the star. To that extent, the atmosphere can be considered as a plane parallel atmosphere. This will not do for a greatly extended atmosphere, such as that of an M supergiant; as to whether it is an appropriate model for a star like the Sun, remind yourself how sharply-defined the limb of the Sun is, and hence how rapidly the atmosphere becomes opaque.

4.2 *Luminosity*

The most important differences between "standard" and "astrophysical" usages are in the meanings of flux and intensity. What we have hitherto called the "radiant flux" Φ of a source of radiation, expressed in watts, is generally called by astronomers, when describing the radiant flux from an entire star, the *luminosity* of the star, and the symbol used is L. While this can be expressed in watts, it is commonly expressed in units of the luminosity of the Sun, which is 3.85×10^{26} watts.

4.3 *Specific Intensity*

Instead of the word "radiance", for which we have hitherto used the symbol L, the name commonly used in the context of stellar atmosphere theory is *specific intensity* and the symbol used is I. In most branches of physics, the word "specific" is used to mean "per unit mass", but that meaning is not intended in the present context. It might be noted that the word "specific" is often omitted, so that what we have hitherto called "radiance" L is now "intensity" I. Note also that this in not the same as what is meant in "standard" usage by "intensity", for which the symbol, in standard usage, is also I. In this and subsequent chapters, I shall always include the word "specific".

The quantity we have in earlier chapters called "radiance" was used to describe the brightness of an extended radiating surface, and the new term "specific intensity" can equally be used in a similar context. More often, however, you may need to imagine yourself embedded somewhere within a stellar atmosphere and you are looking around to see the watts per square metre per steradian arriving at you from various directions.

4.4 *Flux*

The word "flux" in the context of stellar atmosphere theory differs from the "flux" of standard terminology. The symbol used is F, occasionally but not always printed in a special font. Let us imagine a horizontal surface embedded somewhere in a stellar atmosphere. It is being irradiated from below and above, but rather more from below than from above. Let us concentrate our attention for the time being on the radiation that is coming up from below. The rate of arrival of radiant energy per unit area from below from all directions would, in "standard" nomenclature, have been called the irradiance. After passage through the surface, it would be called the exitance. Now, using the nomenclature of stellar atmosphere theory, we call the rate of upward passage of radiant energy per unit area through a horizontal surface within the atmosphere the *upward* or the *outward flux*. The symbol is F_+ (sometimes printed in a special font), and the SI units would be W m^{-2}. Likewise, the rate of passage of radiant energy per unit area from above to below is the *downward* or *inward flux,* F_-. The net upward flux is $F = F_+ - F_-$. If you are standing upright, F_+ is the irradiance of the soles of your feet, while F_- is the irradiance of the top of your head.. If we measure the spherical angle θ from a downwardly-directed z-axis, then, following equation 1.14.5, we have

$$F_+ = \int_0^{2\pi} \int_0^{\pi/2} I(\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi \qquad 4.4.1$$

$$F_- = \int_0^{2\pi} \int_\pi^{\pi/2} I(\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi \qquad 4.4.2$$

$$F = F_+ - F_- = \int_0^{2\pi} \int_0^{\pi} I(\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi, \qquad 4.4.3$$

which is sometimes written for short

$$F = \int_{4\pi} I\cos\theta\, d\omega \text{ or just } \int I\cos\theta\, d\omega \qquad 4.4.4$$

At the surface of the star (if there is such a thing!) $F_- = 0$, so that $F = F_+ = \pi I$. This is the same as equation 1.15.2. At the centre of the star, I is isotropic and $F = 0$.

4.5 *Mean Specific Intensity*

Look around you again from your position somewhere in the middle of a stellar atmosphere. The specific intensity around you is not isotropic. It is quite large in the sky above you, but is much greater if you look towards the hell at your feet. The *mean specific intensity* J around the complete 4π steradians around you is

$$J = \tfrac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I(\theta)\sin\theta\, d\theta\, d\phi \qquad 4.5.1$$

or, for short

$$J = \tfrac{1}{4\pi}\int I d\omega. \qquad 4.5.2$$

At the centre of the star, where the specific intensity is isotropic, $J = I$.

4.6 *Radiation Pressure*

Radiation Pressure. Recall equation 1.18.5 and the conditions for which it is valid. It was derived for isotropic radiation. In the atmosphere, radiation is not isotropic; there is a net flux of radiation outwards. Therefore the radiation density must go inside the integral sign. We can also write the equation in terms of specific intensity, making use of equations 1.15.3 and 1.17.1. The equation for the radiation pressure then becomes

$$P = \frac{1}{c}\int_{4\omega} I\cos^2\theta\, d\omega, \qquad 4.6.1$$

where by now we are used to the abbreviated notation.

If the radiation is isotropic, this is not zero; it is $4\pi/(3c)$. In the expressions for J and for P, the power of $\cos\theta$ is even (0 and 2 respectively) and one can see both physically and mathematically that neither of them is zero for isotropic radiation. One the other hand, the expression for F has an odd power of $\cos\theta$, and it is therefore zero for isotropic radiation, as expected.

4.7 *Other Integrals*

Other integrals occurring in the theory of stellar atmospheres are (using the abbreviated notation)

$$H = \tfrac{1}{4\pi}\int I\cos\theta\, d\omega = F/(4\pi) \qquad 4.7.1$$

$$= 0 \text{ if isotropic} \qquad 4.7.2$$

$$K = \tfrac{1}{4\pi}\int I\cos^2\theta\, d\omega = cP/(4\pi) \qquad 4.7.3$$

$$= J/3 \text{ if isotropic.} \qquad 4.7.4$$

The SI units for F are W m^{-2}. For I, J, H, K they are W m^{-2} sr^{-1}. For P they are Pa.

4.8 *Emission Coefficient*

This is the intensity emitted per unit volume of a gas. In this definition, the word “intensity” (without being preceded by the word “specific”) is intended to mean what is meant in section 1.4. The symbol used for emission coefficient is j, and the SI units are W sr^{-1} m^{-3}.

The *mass emission coefficient* j_m is the intensity per unit mass, and is related to the volume emission coefficient by $j = \rho\, j_m$. Not all authors are equally clear as to whether they are referring to a volume or a mass emission coefficient.