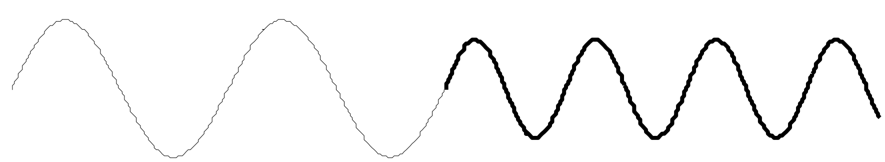
REFLECTION AND TRANSMISSION AT AN INTERFACE

1. *Introduction*

When a ray of light encounters an interface between two media of different refractive indices, some of it is reflected and some is transmitted. This chapter will concern itself with how much is reflected and how much is transmitted. (Unless the media are completely transparent, some of the light will also be *absorbed* - and presumably degraded as heat - but this chapter will concern itself only with what happens at the interface, and not in its passage through either medium.) We shall do this at three levels: Normal incidence; incidence at the Brewster angle (we'll explain what is meant by this); incidence at an arbitrary angle.

2. *Waves in a stretched string*

Before discussing the reflection of light, it will be useful to discuss the following problem. Consider two ropes, one thin and one thick, connected together, and a sinusoidal wave moving from left to right along the ropes:



The speed c of waves in a rope under tension is $c = \sqrt{F/\mu}$, where F is the tension, and μ is the mass per unit length, so the speed and the wavelength are less in the thicker rope. We'll call the speed in the left hand rope c_1 and the speed in the right hand rope c_2. At the boundary ($x = 0$), some of the wave is transmitted, and some is reflected. (I haven't drawn the reflected part in the drawing). We wish to find how much is transmitted and how much is reflected. I'll call the amplitudes of the incident, transmitted and reflected waves 1, T and R respectively, and I'll suppose that the wave is a sinusoidal wave of angular frequency ω. The equations to the incident, transmitted and reflected waves are as follows:

$$y = \cos\omega\left(t - \frac{x}{c_1}\right)$$

$$y = T\cos\omega\left(t - \frac{x}{c_2}\right)$$

$$y = R\cos\omega\left(t + \frac{x}{c_1}\right)$$

To the right of the boundary, the displacement as a function of x and t is

$$y = T\cos\omega\left(t - \frac{x}{c_2}\right)$$

and to the left of the boundary the displacement is

$$y = \cos\omega\left(t - \frac{x}{c_1}\right) + R\cos\omega\left(t + \frac{x}{c_1}\right).$$

At the boundary ($x = 0$), unless the rope breaks these two displacements must be equal, and therefore

$$T = 1 + R. \qquad (1)$$

The x-derivatives (i.e. the slopes) of the ropes are:

To the right of the boundary

$$\frac{\partial y}{\partial x} = \frac{T}{c_2}\sin\omega\left(t - \frac{x}{c_2}\right)$$

and to the left of the boundary

$$\frac{\partial y}{\partial x} = \frac{A}{c_1}\sin\omega\left(t - \frac{x}{c_1}\right) - \frac{AR}{c_1}\sin\omega\left(t + \frac{x}{c_1}\right).$$

Unless there is a kink in the rope at the boundary, these are equal at $x = 0$, and therefore

$$\frac{T}{c_2} = \frac{1}{c_1} - \frac{R}{c_1}. \tag{2}$$

Combining these with equation 1, we obtain

$$T = \frac{2c_2}{c_2 + c_1} \quad \text{and} \quad R = \frac{c_2 - c_1}{c_2 + c_1}. \tag{3}$$

We see that if $c_2 < c_1$, R is negative; that is, there is a phase change at reflection. If $c_2 = c_1$ (i.e. if there is only one sort of rope) there is no reflection (because there is no boundary!).

In the above analysis, we considered a simple sine wave. However, any function, even a nonperiodic function, can be represented by a sum (perhaps an infinite sum) of sinusoidal waves, so the same result will be obtained for any function.

One hopes that energy is conserved, so let's see. The energy in a wave is proportional to the square of its amplitude and, in the case of a vibrating rope, to the mass per unit length. And the rate of transmission of energy is equal to this times the speed. Thus the rate of transmission of energy is proportional to $A^2\mu c$. But $c = \sqrt{F/\mu}$, so that the power is proportional to A^2/c. Thus the incident, transmitted and reflected powers are in the ratio

$$1 \;:\; \frac{4c_1c_2}{(c_1+c_2)^2} \;:\; \frac{(c_1-c_2)^2}{(c_1+c_2)^2}. \tag{4}$$

We see that the sum of the transmitted and reflected powers is equal to the incident power, and all's well with the world.

3. *Light incident normally at a boundary*

The result described by equation (3) for the transmitted and reflected amplitudes is an inevitable consequence of the continuity of displacement and gradient of a wave at a boundary, and is not particularly restricted to waves in a rope. It should be equally applicable to electromagnetic waves moving from one medium to another at normal incidence, and indeed it is verified by measurement. Thus, as with the ropes, the amplitudes of the incident, transmitted and reflected waves are in the ratio

$$1 \;:\; \frac{2c_2}{c_2 + c_1} \;:\; \frac{c_2 - c_1}{c_2 + c_1}. \tag{5}$$

One hopes that energy is conserved, so let's see. The energy stored per unit volume in an electric field in an isotropic medium is $\frac{1}{2}\varepsilon E^2$. The rate of transmission of energy per unit area (i.e. the flux density) is this times the speed of propagation. But $\varepsilon = \frac{1}{\mu_0 c^2}$. (We suppose in the present context that both media are nonmagnetic, so both have permeability μ_0.) Thus we see that the rate of propagation of energy per unit area is proportional to the square of the amplitude and inversely proportional to the speed.

Thus the incident, transmitted and reflected powers are in the ratio

$$1 \quad : \quad \frac{4c_1c_2}{(c_1 + c_2)^2} \quad : \quad \frac{(c_1 - c_2)^2}{(c_1 + c_2)^2} \,. \tag{6}$$

As with the two ropes, the sum of the transmitted and reflected flux densities is equal to the incident flux density, and, once again, all's well with the world.

It may at first glance be surprising that the rate of transmission of energy is *inversely* proportional to the speed. In the case of the ropes, the "slow" rope has a larger mass per unit length. In the case of the electromagnetic field, the "slow" medium has a larger permittivity, so the electric field is having to work the harder.

The speed of light in a medium is inversely proportional to the refractive index, so the amplitude ratios can be expressed as

$$1 \quad : \quad \frac{2n_1}{n_1 + n_2} \quad : \quad \frac{n_1 - n_2}{n_1 + n_2}. \tag{7}$$

We see that there is a phase change on reflection from an optically denser medium.

The flux density ratios can be written as

$$1 \quad : \quad \frac{4n_1n_2}{(n_2 + n_1)^2} \quad : \quad \frac{(n_2 - n_1)^2}{(n_2 + n_1)^2} \,. \tag{8}$$

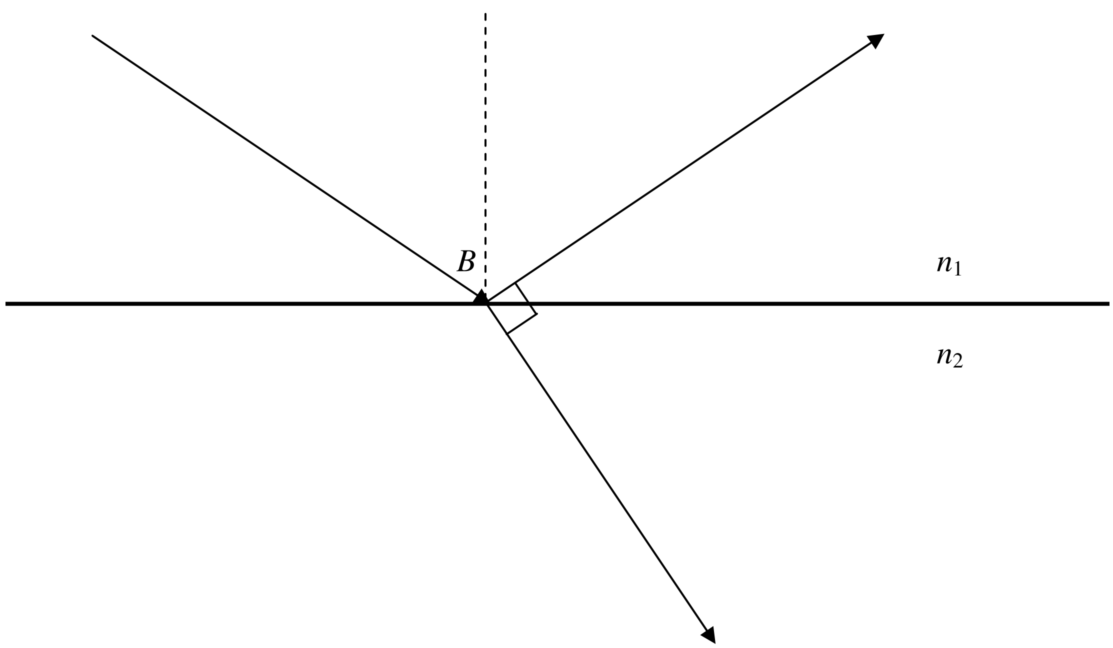
If light is going from air ($n_1 = 1$) to glass ($n_2 = 1.5$), the transmitted amplitude will be 80 percent of the incident amplitude, and the reflected amplitude will be 20 percent of the incident amplitude. The transmitted flux density will be 96 percent of the incident flux density, and the reflected flux density will be 4 percent of the incident flux density.

If $n_2 = n_1$ there will be no reflection at the boundary; in effect there is no boundary. The larva of the midge *Chaoborus*, known as the Phantom Midge, is an aquatic creature whose body has a refractive index equal to the refractive index of water. The picture below shows a photograph of one of them in the water:

Larva of the Phantom Midge, *Chaoborus* sp.

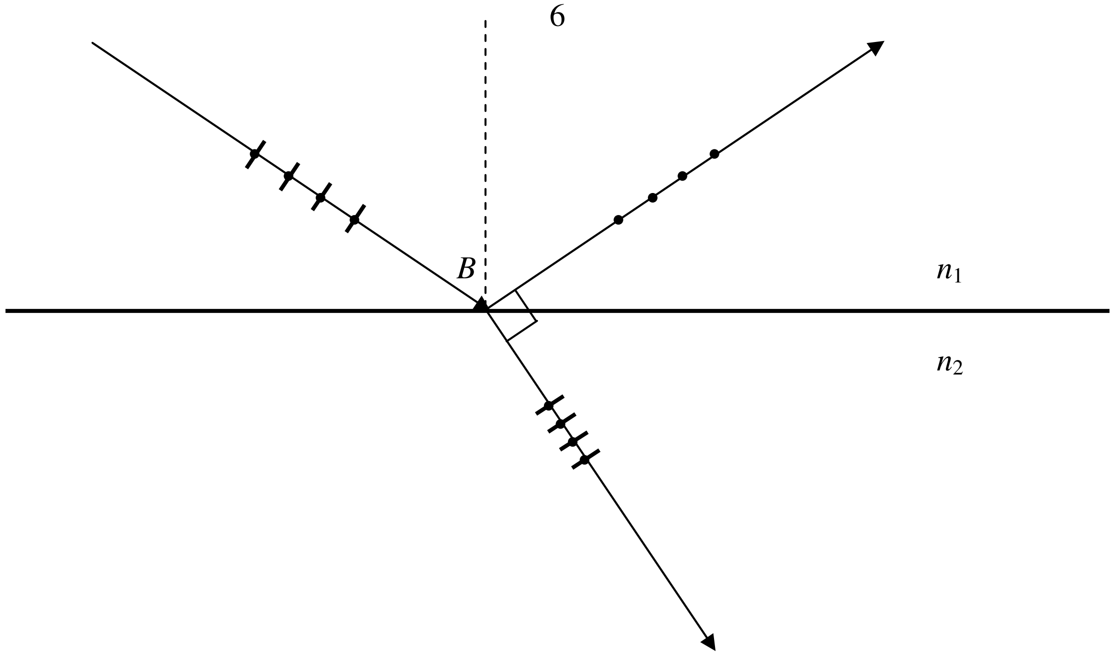
(If you don’t believe me, look it up on the Web.)

4. *Light incident at the Brewster angle*



If a ray of light is incident at an interface between two media in such a manner that the reflected and transmitted rays are at right angles to each other, the angle of incidence, B, is called the *Brewster angle*. A moment’s thought will show that, if the refractive indices are n_1 and n_2, $\tan B = n_2 / n_1$. For example, at an air ($n_1 = 1$) to glass ($n_2 = 1.5$) interface the Brewster angle is 56 degrees.

If a ray of unpolarized light is incident at the Brewster angle, the reflected ray is totally plane-polarized. The is no component of the oscillating electric field that is in the plane



of the paper and at right angles to the direction of propagation of the reflected ray. The transmitted ray, having lost some of the component of the electric field at right angles to the plane of the paper (i.e. the dots) is partially plane polarized.

5. *Electric and magnetic fields at a boundary*

We next want to discuss the reflection and transmission for an arbitrary angle of incidence. Before we can do this it is well to remind ourselves (and this is just a reminder - we don't go into the theory and definitions here) from electromagnetic theory how electric and magnetic fields behave at a boundary between two media.

If an electric field is incident normally at the boundary between two media, E is larger in the medium with the smaller permittivity, whereas D is continuous. Likewise, if a magnetic field is incident normally at the boundary between two media, H is smaller in the medium with the higher permeability, whereas B is continuous.

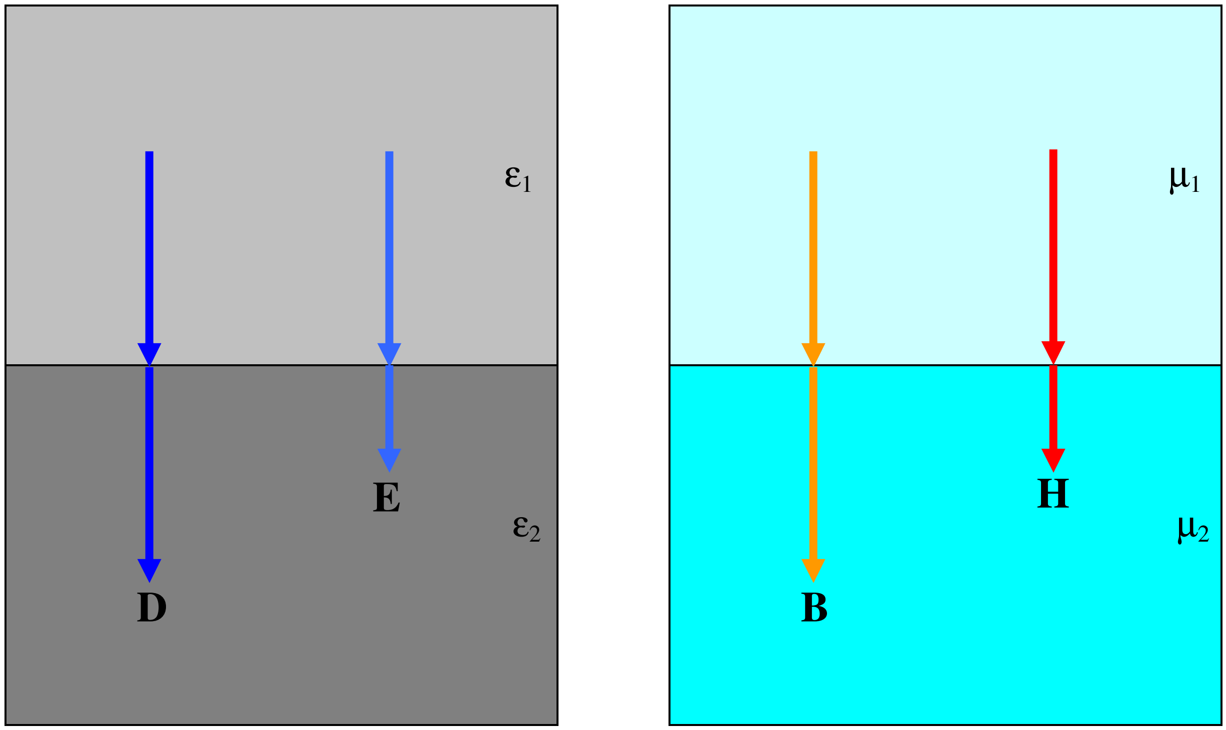
That is:

D_{perp} and B_{perp} are continuous across a boundary.

E_{perp} is inversely proportional to ε.

H_{perp} is inversely proportional to μ.

See the drawing below.

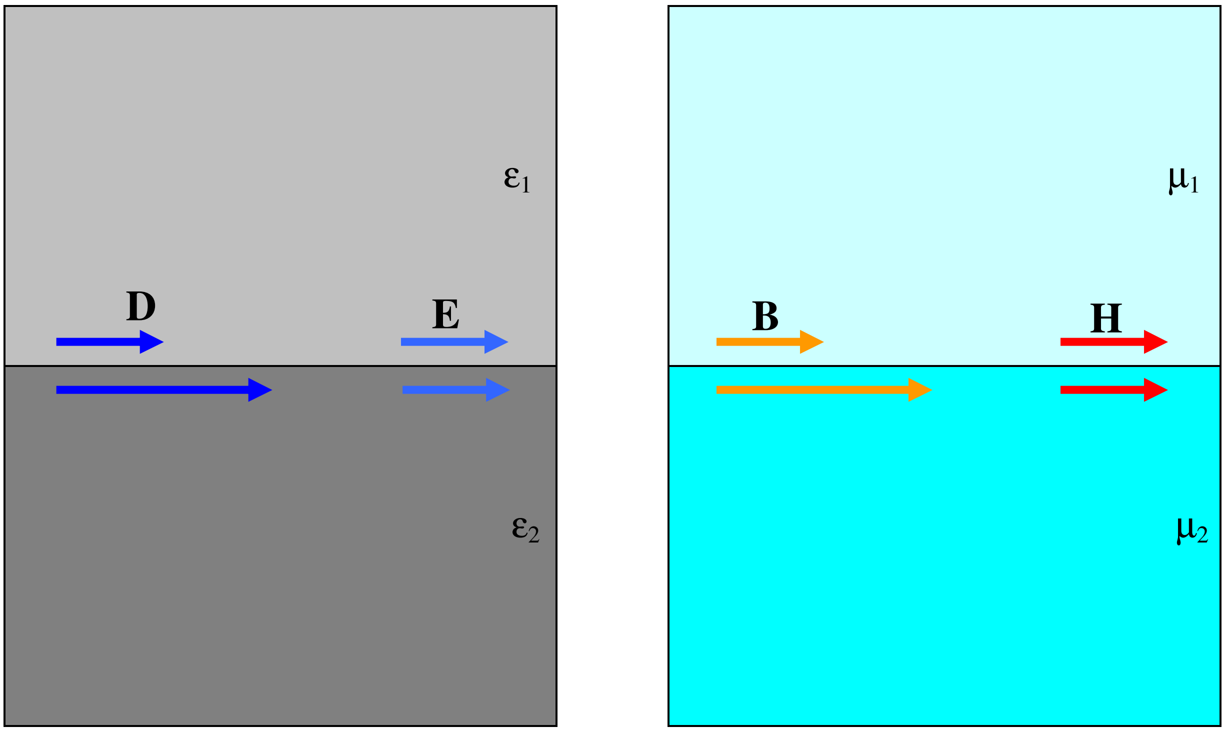

ε1
E
ε2
D
μ1
H
μ2
B

For fields parallel to a boundary, however, the situation is:

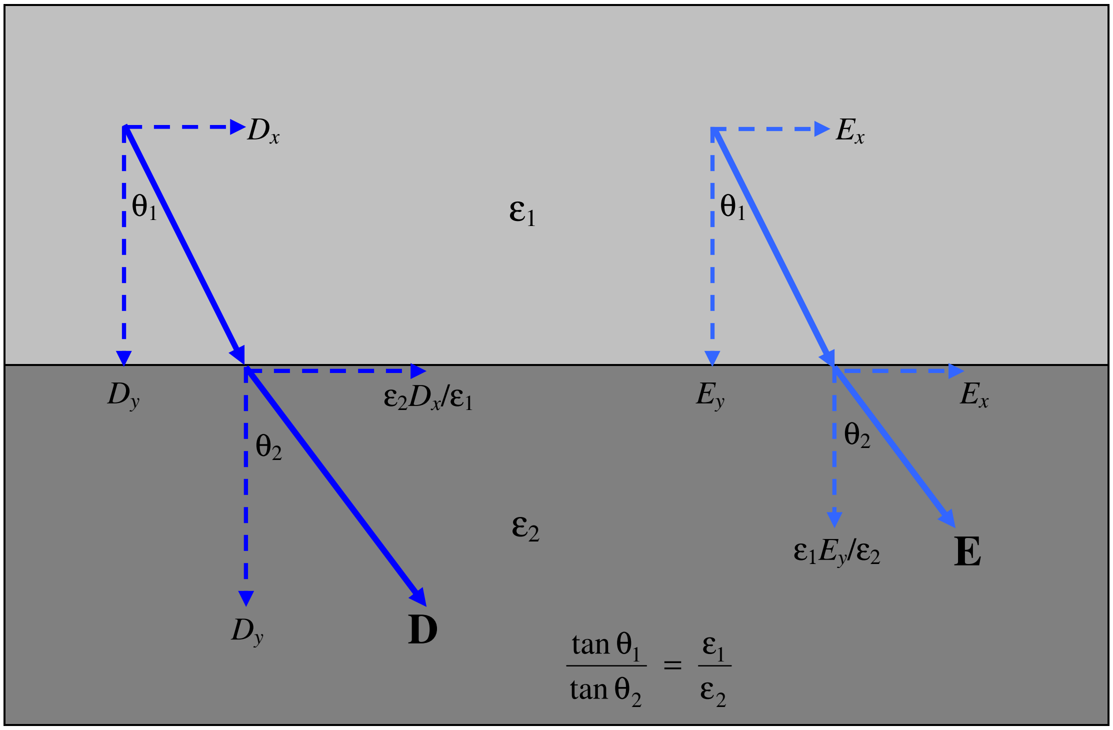
E_{tang} and H_{tang} are continuous across a boundary.

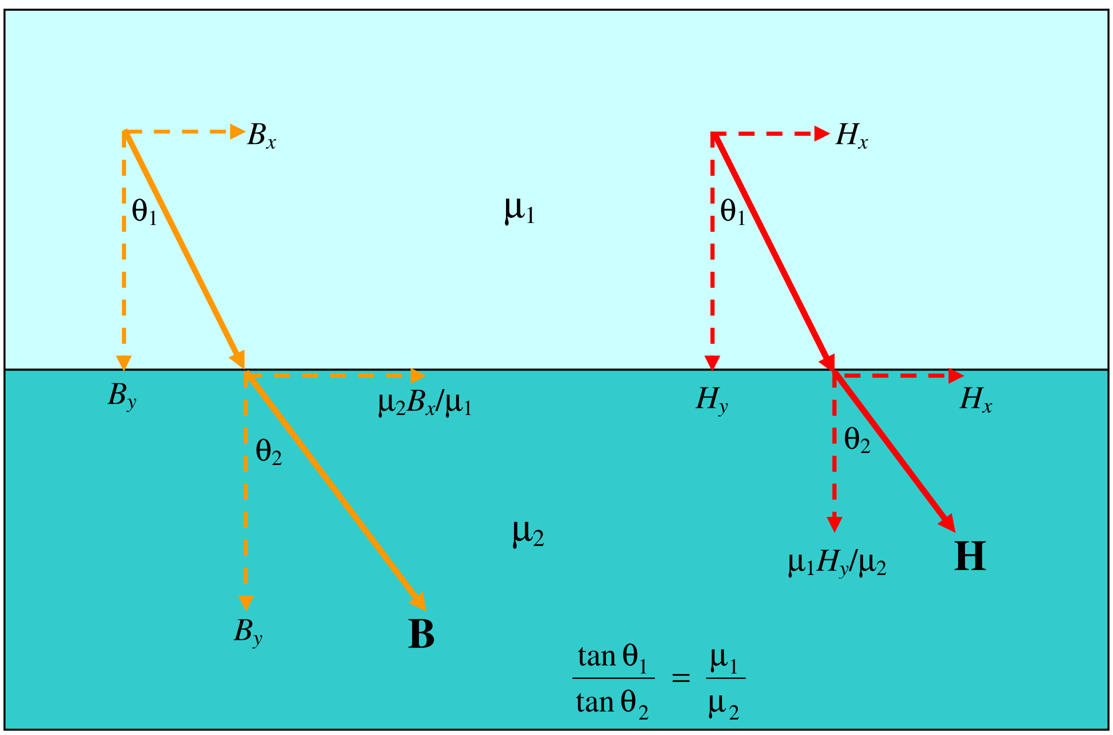
D_{tang} is proportional to ε.

B_{tang} is proportional to μ.



These things are assumed known from courses in electromagnetism. It may be asked what happens if a field is neither perpendicular to nor tangential to a boundary. We do not especially need to know that in discussing reflection of light at a boundary, because we shall be resolving any fields into their perpendicular and tangential components, but it is a reasonable question to ask, so for completeness the answers are given in the drawings below.


D_x
θ_1
ε_1
E_x
θ_1
D_y
$\varepsilon_2 D_x/\varepsilon_1$
E_y
E_x
θ_2
θ_2
ε_2
$\varepsilon_1 E_y/\varepsilon_2$
E
D_y
D
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$


B_x
H_x
θ_1
μ_1
θ_1
B_y
$\mu_2 B_x/\mu_1$
H_y
H_x
θ_2
θ_2
μ_2
$\mu_1 H_y/\mu_2$
H
B_y
B
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$$

6. *Impedance*

We need to remind ourselves of one other thing from electromagnetic theory before we can proceed, namely the meaning of *impedance* in the context of electromagnetic wave propagation. The impedance Z is merely the ratio E/H of the electric to the magnetic field. The SI units of E and H are V/m and A/m respectively, so the SI units of Z are V/A, or ohms, Ω. We are now going to see if we can express the impedance in terms of the permittivity and permeability of the medium in which an electromagnetic wave is travelling.

Maxwell's equations are

$$\nabla \bullet \mathbf{D} = \rho$$
$$\nabla \bullet \mathbf{B} = 0.$$
$$\nabla \times \mathbf{H} = \dot{\mathbf{D}} + \mathbf{J}.$$
$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}.$$

In an isotropic, homogeneous, nonconducting, uncharged medium (such as glass, for example), the equations become:

$$\nabla \bullet \mathbf{E} = 0$$
$$\nabla \bullet \mathbf{H} = 0$$
$$\nabla \times \mathbf{H} = \varepsilon \dot{\mathbf{E}}.$$
$$\nabla \times \mathbf{E} = -\mu \dot{\mathbf{H}}.$$

If you eliminate **H** from these equations, you get

$$\nabla^2 \mathbf{E} = \varepsilon\mu \ddot{\mathbf{E}},$$

which describes an electric wave of speed $1/\sqrt{\varepsilon\mu}$. In free space this becomes $1/\sqrt{\varepsilon_0\mu_0}$, which is 2.998×10^8 m s^{-1}.

The ratio of the speeds in two media is $\frac{\upsilon_1}{\upsilon_2} = \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}$, and if, as is often the case, the two permeabilities are equal (to μ_0), then $\frac{\upsilon_1}{\upsilon_2} = \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$. In particular, if you compare one medium with a vacuum, you get: $n = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$.

Light is a high-frequency electromagnetic wave. When a dielectric medium is subject to a high frequency field, the polarization (and hence D) cannot keep up with the electric field E. D lags behind E. This can be described mathematically by ascribing a complex value to the permittivity. The amount of lag depends, unsurprisingly, on the frequency - i.e. on the colour - and so the permittivity and hence the refractive index depends on the wavelength of the light. This is *dispersion*.

If instead you eliminate $\mathbf{E}$ from Maxwell's equations, you get

$$\nabla^2\mathbf{H} = \varepsilon\mu\ddot{\mathbf{H}}.$$

This is a magnetic wave of the same speed.

If you eliminate the time between $\nabla^2\mathbf{E} = \varepsilon\mu\ddot{\mathbf{E}}$ and $\nabla^2\mathbf{H} = \varepsilon\mu\ddot{\mathbf{H}}$, you find that

$\dfrac{E}{H} = \sqrt{\dfrac{\mu}{\varepsilon}}$, which, in free space, has the value $\sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 377\ \Omega$, which is the impedance of free space. In an appropriate context I may use the symbol Z_0 to denote the impedance of free space, and the symbol Z to denote the impedance of some other medium.

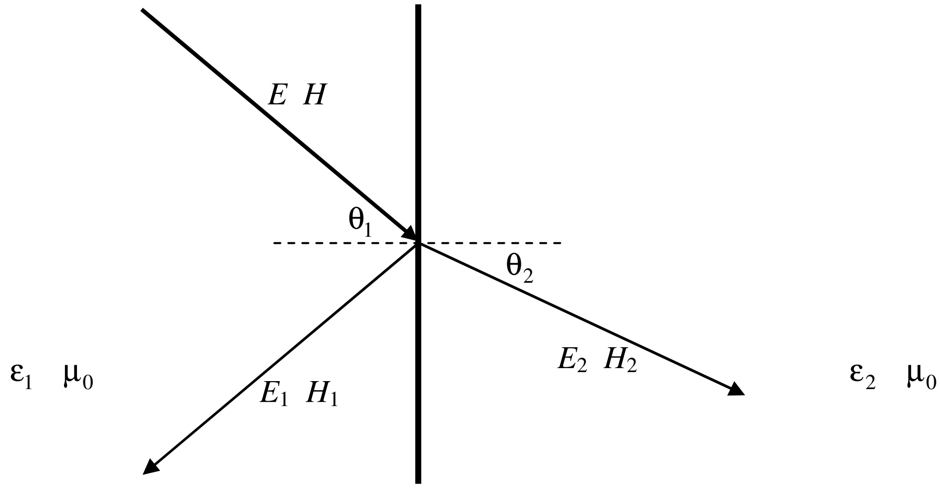
The ratio of the impedances in two media is $\dfrac{Z_1}{Z_2} = \sqrt{\dfrac{\varepsilon_2\mu_1}{\varepsilon_1\mu_2}}$, and if, as is often the case, the two permeabilities are equal (to μ_0), then $\dfrac{Z_1}{Z_2} = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = \dfrac{n_2}{n_1} = \dfrac{\upsilon_1}{\upsilon_2}$.

We shall be using this result in what follows.

7. *Incidence at an arbitrary angle.*

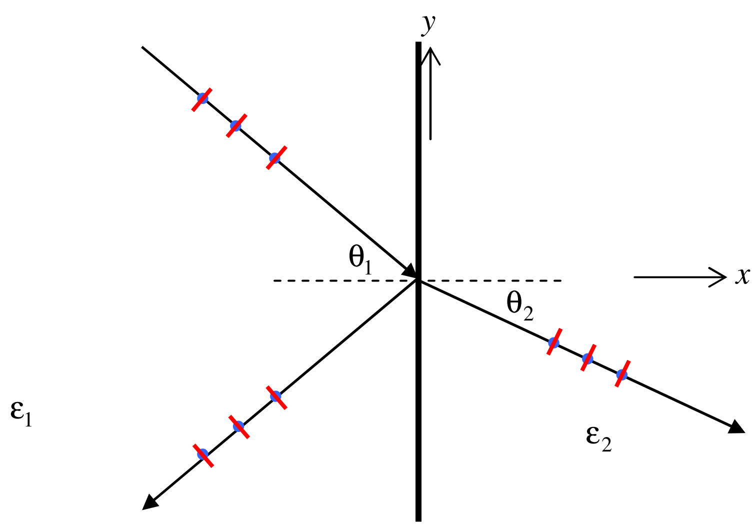
In Section 4 (Incidence at the Brewster Angle) it became clear that the reflection of light polarized in the plane of incidence was different from the reflection of plane polarized light polarized at right angles to the plane incidence. Therefore it makes sense, in this section, to consider the two planes of polarization separately. I shall suppose that both media are isotropic (i.e. not birefringent).

In the following discussion, we'll suppose that light is travelling from a medium of permittivity ε_1 to a medium of greater permittivity ε_2. Both permeabilities are equal, and close to μ_0. The electric and magnetic fields of the *incident* wave will be denoted by E and H. The electric and magnetic fields of the *reflected* wave will be denoted by E_1 and H_1. The electric and magnetic fields of the *transmitted* wave will be denoted by E_2 and H_2. (And in case you are wondering, by H I mean H, and by B I mean B.)



Coordinates: x to the right. y upwards z towards you.

We'll start by supposing that the **incident light is plane polarized with the electric field perpendicular (senkrecht) to the plane of incidence**. That is, the electric field has only a z-component. The oscillating electric field E is indicated by blue dots, and the magnetic field H by red dashes in the drawing below.



The boundary conditions are:
For the tangential (z) component of **E**

$$E + E_1 = E_2. \tag{9}$$

For the tangential (y) component of **H**

$$(H - H_1)\cos\theta_1 = H_2\cos\theta_2 . \tag{10}$$

That is,

$$\frac{(E - E_1)}{Z_1}\cos\theta_1 = \frac{E_2}{Z_2}\cos\theta_2 , \tag{11}$$

or

$$n_1(E - E_1)\cos\theta_1 = n_2E_2\cos\theta_2 . \tag{12}$$

Eliminate E_2 between equations (9) and (12):

Reflected amplitude:

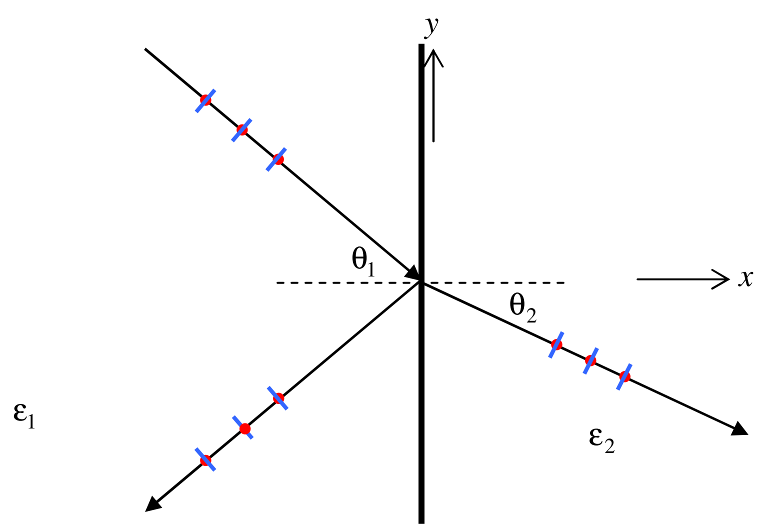
$$\frac{E_1}{E} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2} . \tag{13}$$

Use equation (9):

Transmitted amplitude

$$\frac{E_2}{E} = \frac{2n_2\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2} . \tag{14}$$

Now we'll supposing that the **incident light is plane polarized with the electric field parallel to the plane of incidence**. This, it is the magnetic field that has only a z-component. The oscillating electric field E is indicated by blue dashes, and the magnetic field H by red dots in the drawing below.



The boundary conditions are:

For the tangential (z) component of **H**

$$H + H_1 = H_2 .$$

That is: $$\frac{E + E_1}{Z_1} = \frac{E_2}{Z_2} \quad \text{or} \quad n_1(E + E_1) = n_2 E_2. \tag{15}$$

For the tangential (y) component of **E**

$$(E - E_1)\cos\theta_1 = E_2 \cos\theta_2 . \tag{16}$$

Eliminate E_2 between equations (15) and (16):

Reflected amplitude: $$\frac{E_1}{E} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}. \tag{17}$$

Use equation (15):

Transmitted amplitude $$\frac{E_2}{E} = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2}. \tag{18}$$

These are the *Fresnel Equations*, gathered together below:

Perpendicular (Senkrecht)

Reflected amplitude: $$\frac{E_1}{E} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}.$$

Transmitted amplitude $$\frac{E_2}{E} = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2}.$$

Parallel

Reflected amplitude: $$\frac{E_1}{E} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}.$$

Transmitted amplitude $$\frac{E_2}{E} = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2}.$$

They evidently depend only on the *ratio* of the refractive indices (i.e. the refractive index of one medium relative to that of the other). If we write $n = n_2 / n_1$, the equations become

Perpendicular (Senkrecht)

Reflected amplitude: $$\frac{E_1}{E} = \frac{\cos\theta_1 - n\cos\theta_2}{\cos\theta_1 + n\cos\theta_2}.$$

Transmitted amplitude $$\frac{E_2}{E} = \frac{2\cos\theta_1}{\cos\theta_1 + n\cos\theta_2}.$$

Parallel

Reflected amplitude: $$\frac{E_1}{E} = \frac{n\cos\theta_1 - \cos\theta_2}{n\cos\theta_1 + \cos\theta_2}.$$

Transmitted amplitude $$\frac{E_2}{E} = \frac{2\cos\theta_1}{n\cos\theta_1 + \cos\theta_2}.$$

For normal incidence, the ratios for the senkrecht component become $\frac{1-n}{1+n}$ and $\frac{2}{1+n}$ as expected. The ratios for the parallel component, however, become $\frac{n-1}{n+1}$ and $\frac{2}{1+n}$, apparently predicting no phase change at external reflection for the parallel component. This is only apparent, however, and the explanation for the apparent anomaly is given on pp. 20-24.

It will be noted that n, θ_1, θ_2 are also related by Snell's law: $\sin\theta_1 = n\sin\theta_2$, so that we can eliminate *n* from Fresnel's equations in order to express them in terms of the angles of incidence and refraction only. If this is done we obtain:

Perpendicular (Senkrecht)

Reflected amplitude: $$\frac{E_1}{E} = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}.$$

Transmitted amplitude: $$\frac{E_2}{E} = \frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1 + \theta_2)} = \frac{2}{1 + \frac{\tan\theta_1}{\tan\theta_2}}.$$

Parallel

Reflected amplitude: $$\frac{E_1}{E} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}.$$

Transmitted amplitude: $$\frac{E_2}{E} = \frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2)}.$$

In perhaps the most useful form of all, we could eliminate θ_2 from the Fresnel equations and hence obtain them as functions of θ_1 and n only. This will enable us easily to calculate the reflected and transmitted amplitudes in terms of the angle of incidence. Thus:

Perpendicular (Senkrecht)

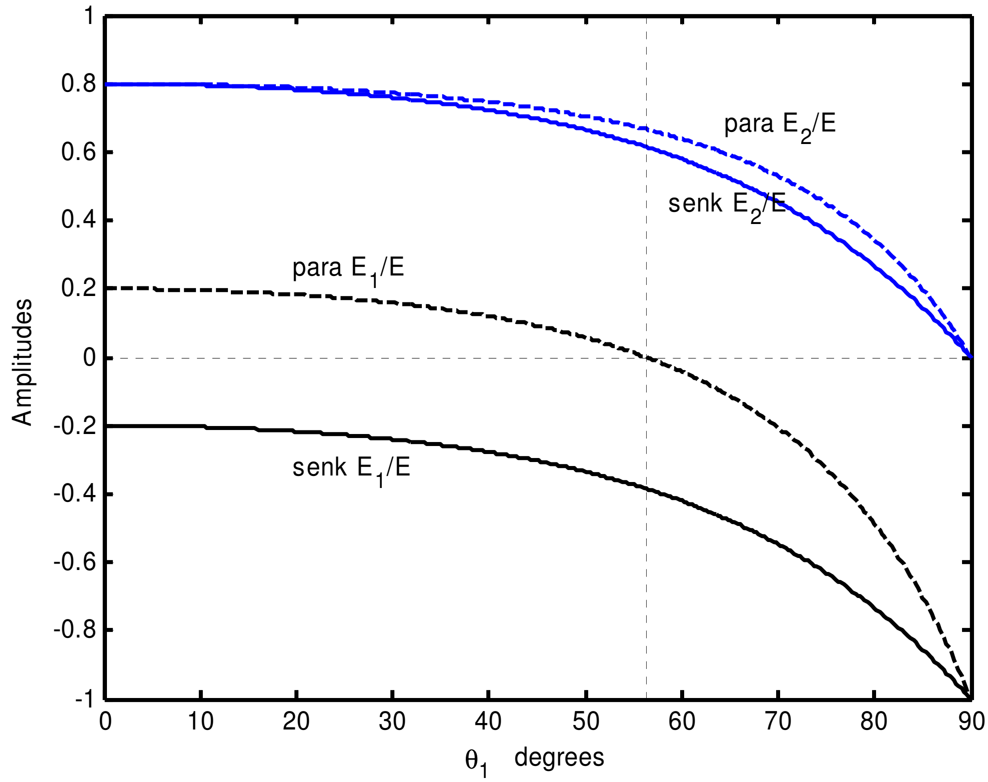
Reflected amplitude: $$\frac{E_1}{E} = -\frac{\left(n^2 - \sin^2\theta_1\right)^{1/2} - \cos\theta_1}{\left(n^2 - \sin^2\theta_1\right)^{1/2} + \cos\theta_1}.$$

Transmitted amplitude: $$\frac{E_2}{E} = \frac{2\cos\theta_1}{(n^2 - \sin^2\theta_1)^{1/2} + \cos\theta_1}.$$

Parallel

Reflected amplitude: $$\frac{E_1}{E} = \frac{n^2\cos\theta_1 - (n^2 - \sin^2\theta_1)^{1/2}}{n^2\cos\theta_1 + (n^2 - \sin^2\theta_1)^{1/2}}.$$

Transmitted amplitude: $$\frac{E_2}{E} = \frac{2n\cos\theta_1}{n^2\cos\theta_1 + (n^2 - \sin^2\theta_1)^{1/2}}.$$



Black curves are the amplitudes of the reflected waves.
Blue curves are the amplitudes of the transmitted waves.

Continuous curves are for senkrecht (perpendicular) waves
Dashed curves are for parallel waves.

Negative values show where there is a 180º phase shift on reflection. Notice that, at the Brewster angle (about 56º), none of the parallel component is reflected.

At 90º (grazing incidence) no light is transmitted; it is all reflected, but with a phase change (negative amplitude).

Energy considerations

Recall that for the parallel component, the incident, reflected and transmitted amplitudes are in the ratio

$$E \;:\; E_1 \;:\; E_2 = 1 \;:\; -\frac{\left(n^2 - \sin^2\theta_1\right)^{1/2} - \cos\theta_1}{\left(n^2 - \sin^2\theta_1\right)^{1/2} + \cos\theta_1} \;:\; \frac{2\cos\theta_1}{\left(n^2 - \sin^2\theta_1\right)^{1/2} + \cos\theta_1}$$

and for the senkrecht component they are in the ratio

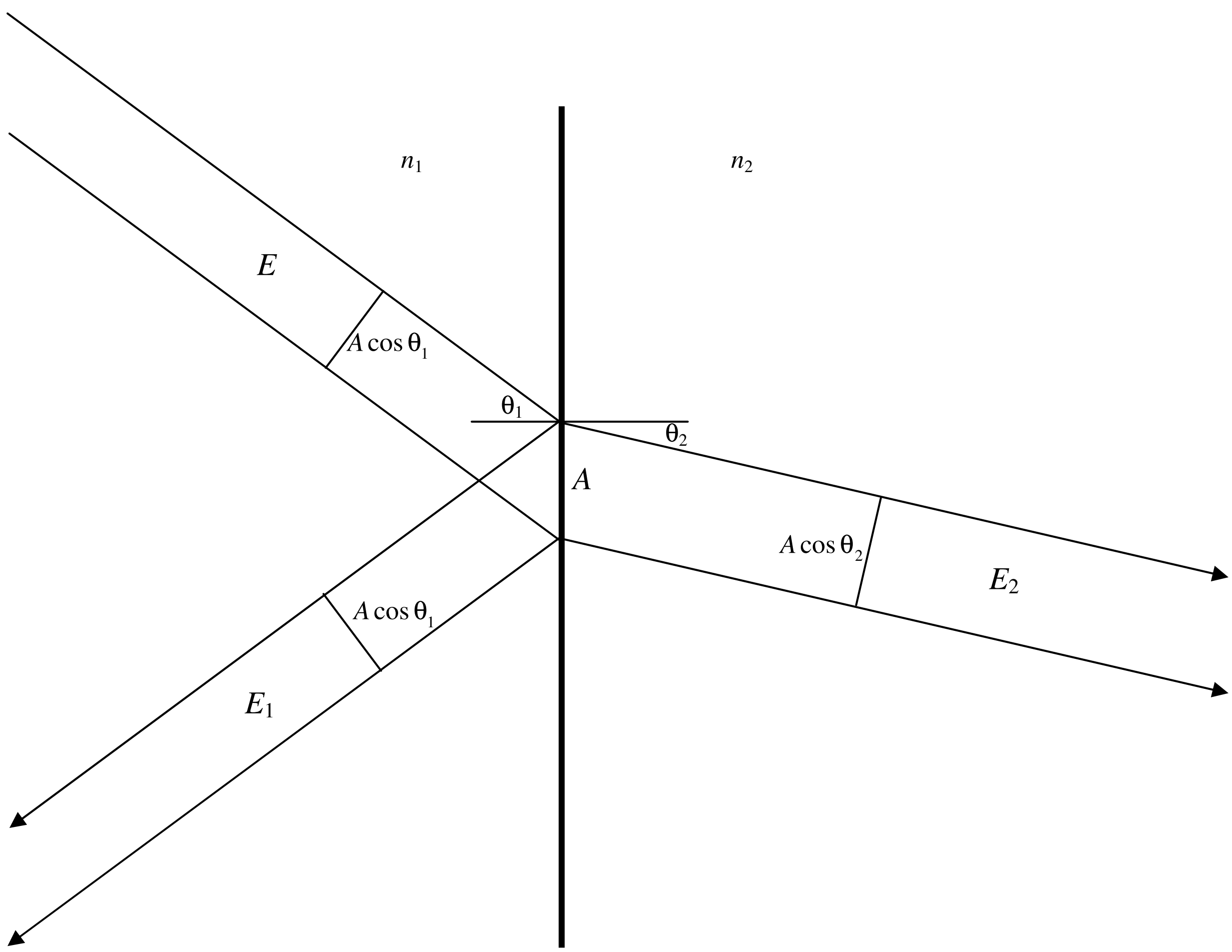
$$E \ : \ E_1 \ : \ E_2 = 1 \ : \ \frac{n^2\cos\theta_1 - \left(n^2 - \sin^2\theta_1\right)^{1/2}}{n^2\cos\theta_1 + \left(n^2 - \sin^2\theta_1\right)^{1/2}} \ : \ \frac{2n\cos\theta_1}{n^2\cos\theta_1 + \left(n^2 - \sin^2\theta_1\right)^{1/2}}$$

.

(Here $n = n_2 / n_1$.)

Suppose that the incident light strikes the interface in an area A. That means that the incident and reflected light are each in beams of cross-sectional area $A\cos\theta_1$, and the transmitted light is in a beam of cross-sectional area A_2. We are going to calculate the ratio $P \ : \ P_1 \ : \ P_2$ of the rate of transmission of energy (power) in each beam; and if we do our algebra correctly, we should find that $P_1 + P_2 = P$.

Recall that the energy per unit volume in an electric field is proportional to εE^2, where ε, the permittivity, is proportional to the square of the refractive index. The power transmitted by each beam is proportional to the energy per unit volume, times the speed of transmission (which is inversely proportional to the refractive index), and to the cross-section area of the beam.



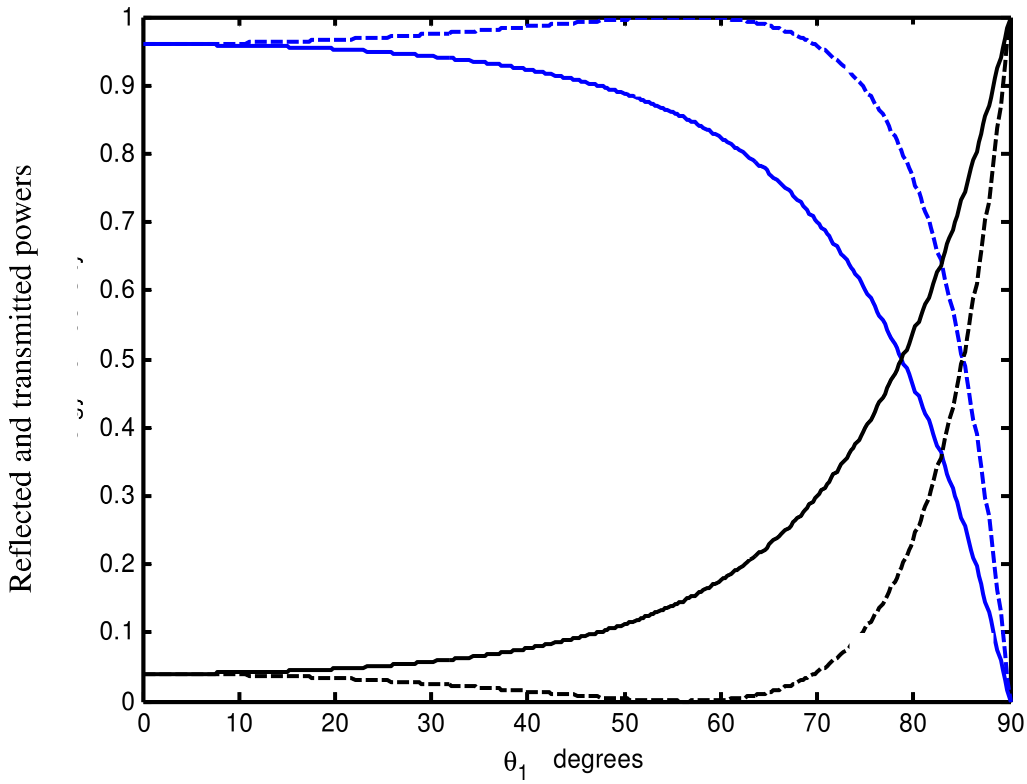
Therefore, for the parallel component and for the senkrecht component,

$$P \;:\; P_1 \;:\; P_2 = n_1 E^2 \cos\theta_1 \;:\; n_1 E_1^2 \cos\theta_1 \;:\; n_2 E_2^2 \cos\theta_2$$

Normalizing this expression so that $P = 1$, we obtain

$$P \;:\; P_1 \;:\; P_2 = 1 \;:\; \left(\frac{E_1}{E}\right)^2 \;:\; n\left(\frac{E_2}{E}\right)^2 \frac{\cos\theta_2}{\cos\theta_1}.$$

These are shown below for $n = 1.5$, and indeed $P_1 + P_2 = P$ for each component, and energy is conserved.

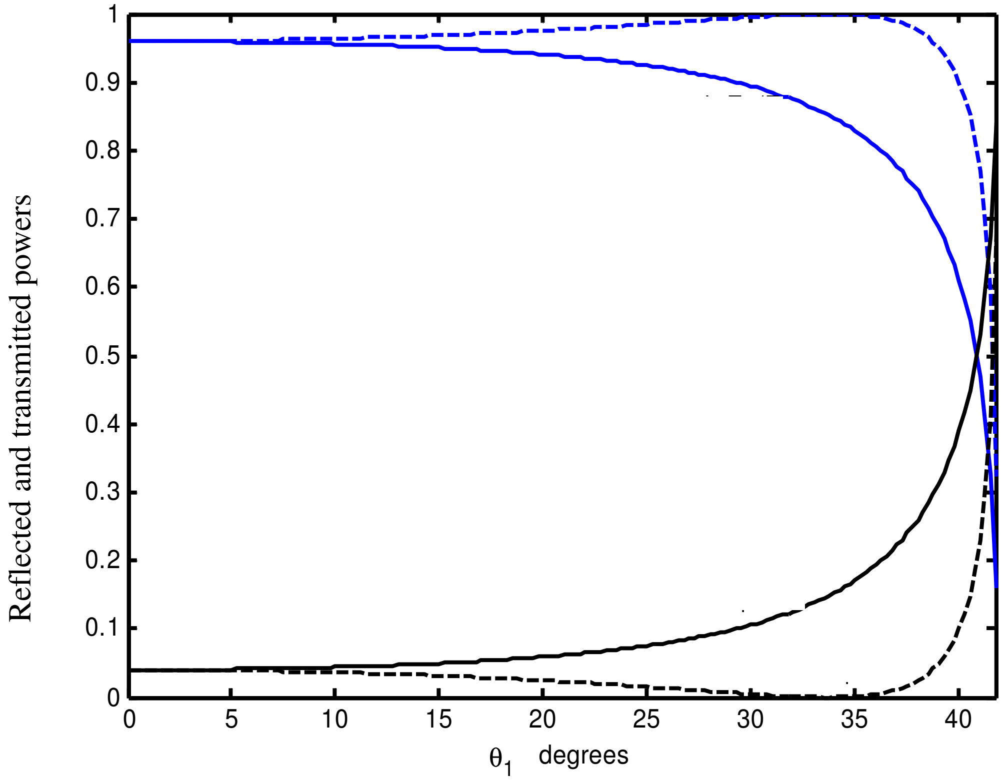


Notice that at grazing incidence we have total external reflection.

Black curves are the reflection coefficients of the reflected waves.
Blue curves are the transmission coefficients of the transmitted waves.

Continuous curves are for senkrecht (perpendicular) waves
Dashed curves are for parallel waves.

At the Brewster angle no parallel waves are reflected.
For light going from $n_1 = 1.5$ to $n_2 = 1$:

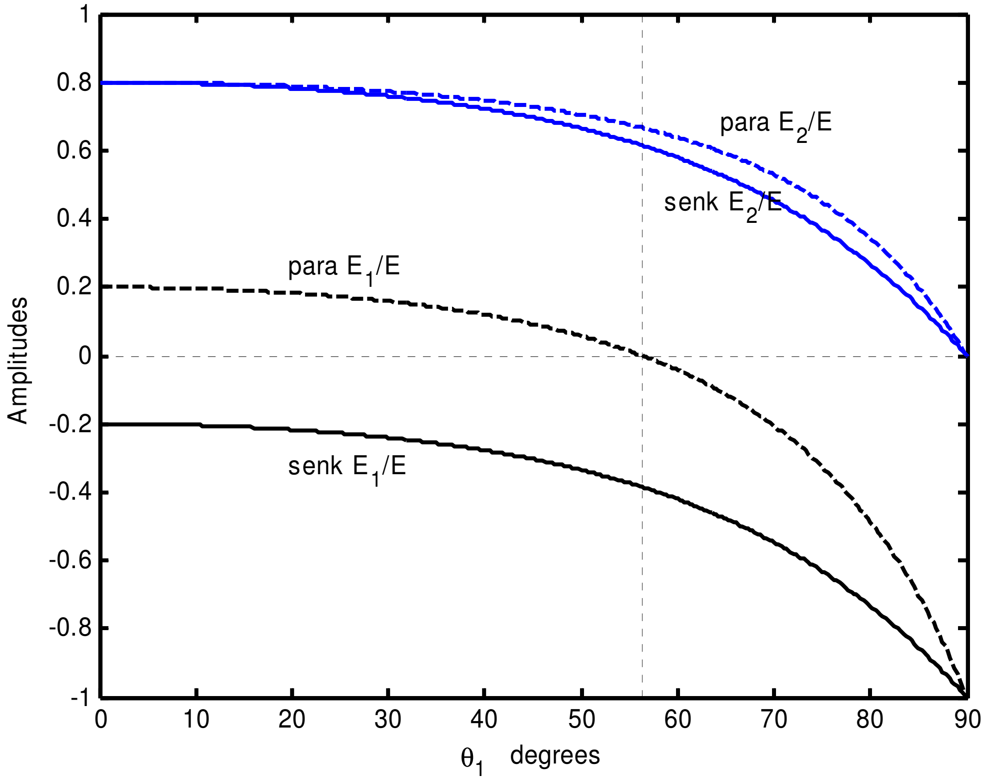


Black curves are the reflection coefficients of the reflected waves.
Blue curves are the transmission coefficients of the transmitted waves.

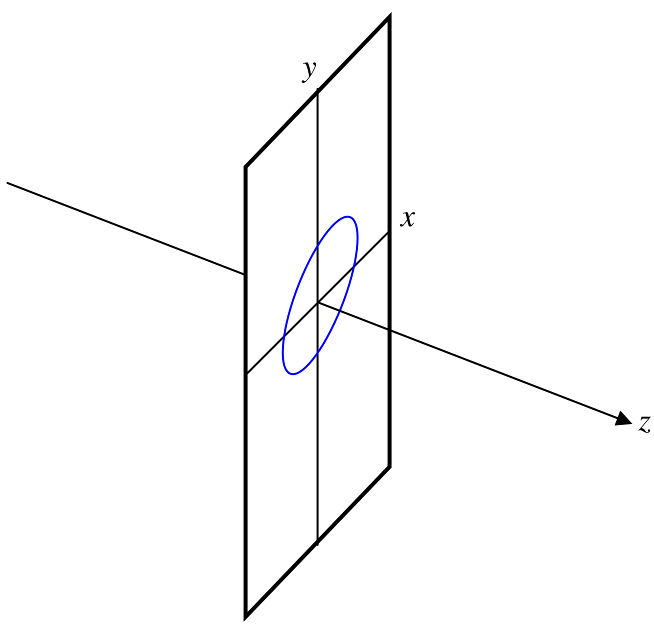
Continuous curves are for senkrecht (perpendicular) waves
Dashed curves are for parallel waves.

For angles of incidence greater than 42 degrees (the critical angle for total internal reflection) all light is reflected. The phase of this totally reflected light is something that we have not yet discussed.

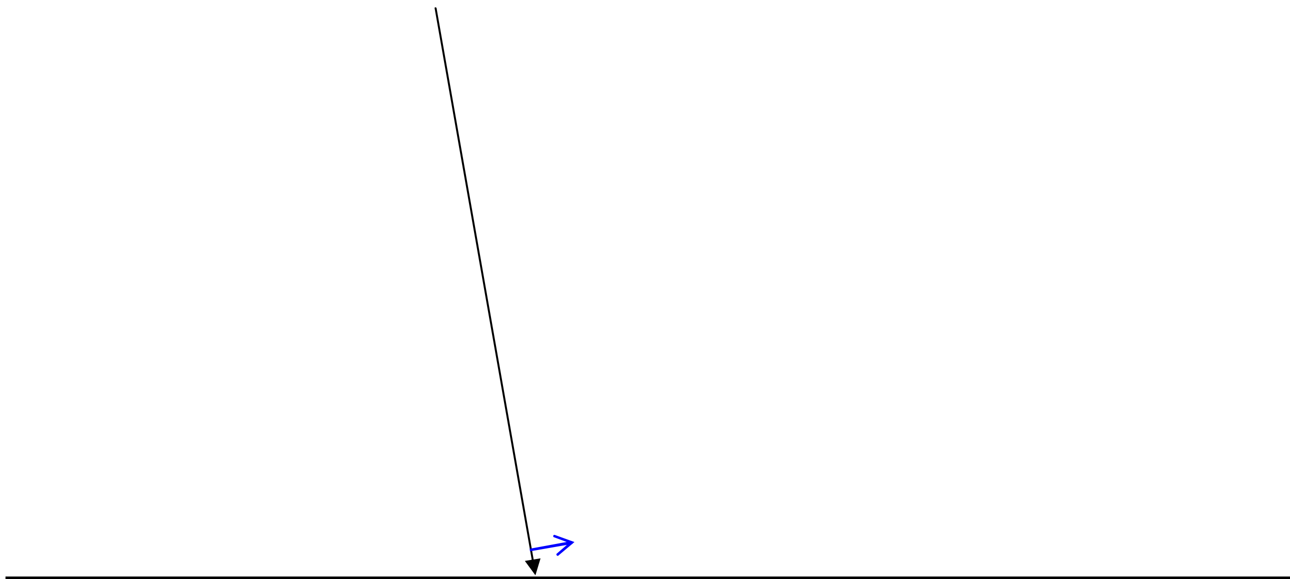
I return now to external reflection and to the graphs, repeated below, which show the reflected and transmitted amplitudes of the parallel and senkrecht components. The blue curves show the transmitted amplitudes, and there is no problem with them. The amplitudes are all positive, meaning that the transmitted waves have no phase change at the boundary. My students pointed out an apparent paradox with the dashed black curve, which is the reflected amplitude of the parallel component. It is positive, indicating (apparently) no phase change, even at normal incidence - and yet we know that there must be a phase change for reflected light at normal incidence. My students demanded (and rightly so) an explanation. The apparent anomaly was also noted on p.15. Following the diagram is the solution that I offer.



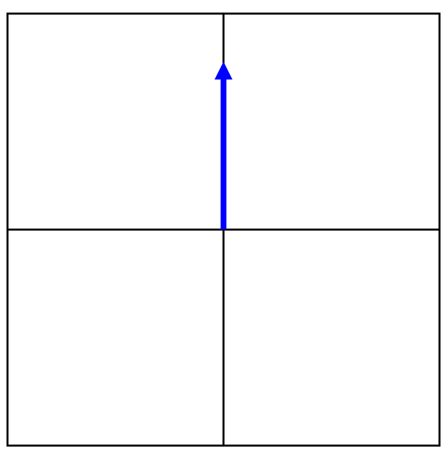
When we describe the state of polarization of light, whether, linear, circular or elliptical, we refer for convenience and of necessity to a coordinate system in which the *z*-axis is in the direction of the ray, and the *xy*-plane is perpendicular to it. The observer is supposed to be on the positive *z*-axis looking towards the source of light:



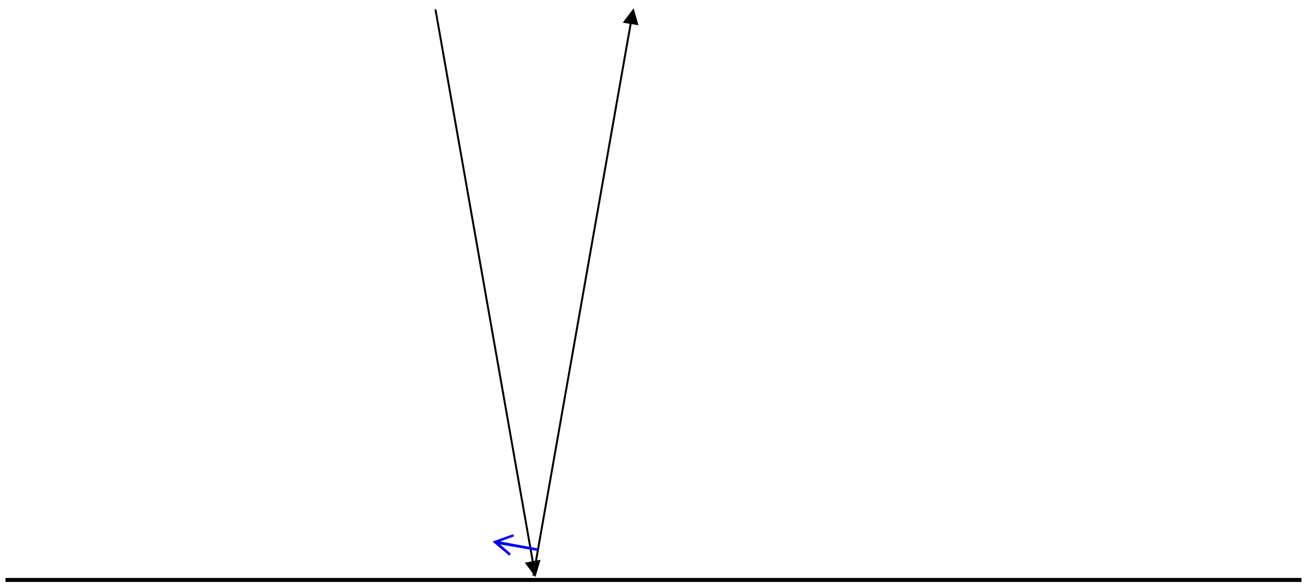
Consider a ray coming down at a steep angle to a water surface. Suppose at some instant of time the electric vector just above the surface is as shown by the little blue arrow below.



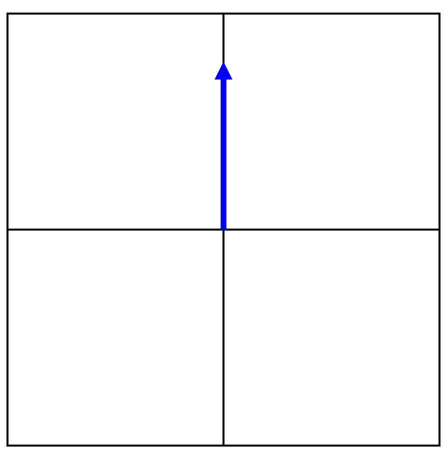
What does our observer (who is underneath the water) see, and how does he describe the state of polarization? This is what he sees:



Now the light is reflected, the observer changes his position, and he looks down on the water from above.



And this is what he sees:



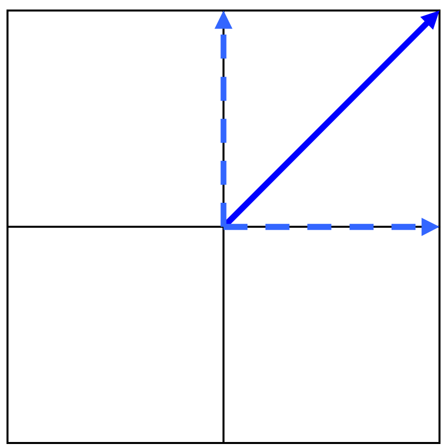
And so there has been no phase change.

Or has there?

One might say that there has been a phase change, but it looks as though there hasn't been. In effect, before and after, we are referring the situation to two reference frames, one of which is the mirror image of the other.

You will see that this apparent paradox does not arise with the senkrecht component.

We have hitherto considered the reflection and transmission of light that was initially plane polarized either parallel to the plane of incidence, or perpendicular (senkrecht) to it. Suppose that the incident light is plane polarized in a direction 45° to the parallel and senkrecht planes. We can resolve it into parallel and senkrecht components, each of amplitude $E/\sqrt{2}$. We suppose that the angle of incidence is θ_1, and the angle of refraction, which is easily calculated from Snell's Law, is θ_2.



After reflection, the amplitudes of the parallel component will be $\frac{E}{\sqrt{2}}\times\frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)}$ and the amplitude of the senkrecht component will be $-\frac{E}{\sqrt{2}}\times\frac{\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)}$.

From these we can calculate the resultant amplitude of the reflected wave as well as its polarization direction (which is quite different from the plane of polarization of the incident wave.)

The transmitted light will have a parallel component of amplitude

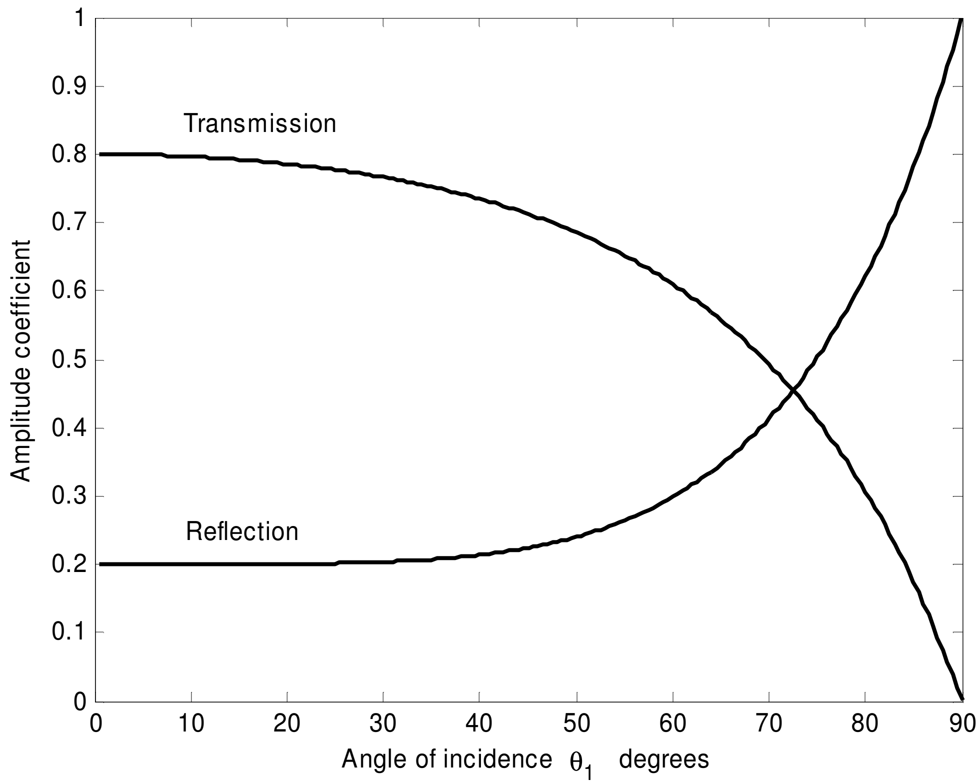
$$\frac{E}{\sqrt{2}}\times\frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)}$$

and a senkrecht component of amplitude

$$\frac{E}{\sqrt{2}} \times \frac{2}{1+\frac{\tan\theta_1}{\tan\theta_2}} .$$

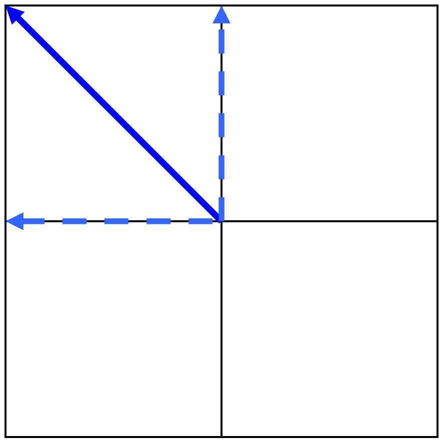
From these we can calculate the resultant amplitude of the transmitted wave as well as its polarization direction (which, as for the reflected wave, is in a different plane from the plane of polarization of the incident wave.)

We show here the magnitudes (without regard to sign) of the amplitude reflection and transmission coefficients, and the polarization directions for the reflected and transmitted wave, as a function of angle of incidence θ_1, assuming $n = n_2 / n_1 = 1.5$.

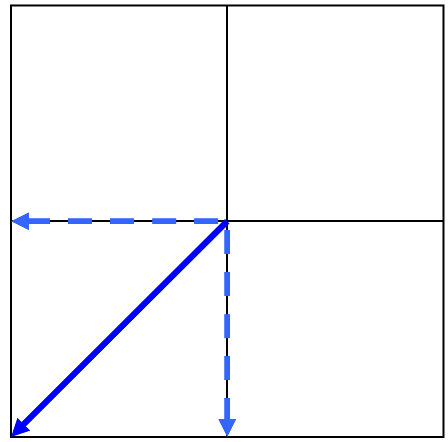


At grazing incidence $\theta_1 = 90°$, all the light is reflected. Although it has no particular significance, we note that, for $n = 1.5$, the reflection and transmission amplitude coefficients are equal (to 0.4544) for an angle of incidence equal to 72º.464. Except for normal and grazing incidence, the reflection and transmission amplitude coefficients do not add exactly to one. While there is a requirement for energy to be conserved, there is no similar requirement for the amplitudes.

As the angle of incidence goes from zero (normal incidence) to 90º (grazing incidence), the plane of polarization of the reflected wave goes from



to



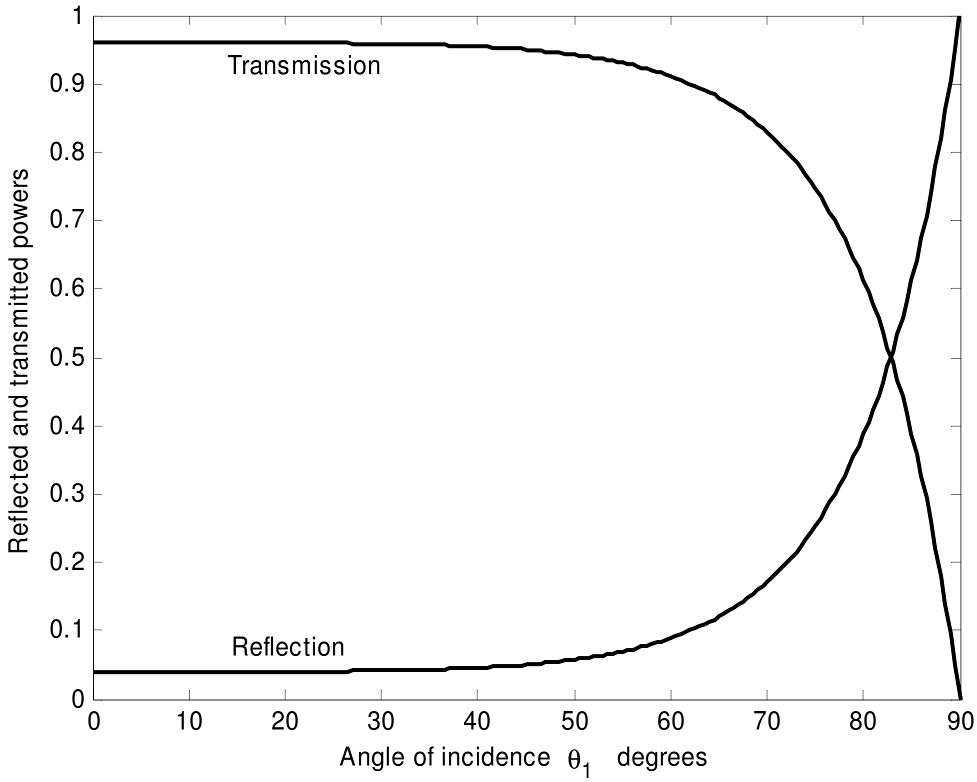
Note that, for normal incidence, the reflected wave has a phase change for the senkrecht component, but (apparently) not for the parallel component, as explained above.

The plane of polarization of the transmitted moves slightly from the initial 45º to 56º.6 (the Brewster angle) at grazing incidence, although this has little significance since no light is transmitted at grazing incidence.

As described on 17-19, if the incident, reflected and transmitted amplitudes are in the ratio $E : E_1 : E_2$, and the corresponding powers are in the ratio $P : P_1 : P_2$, then

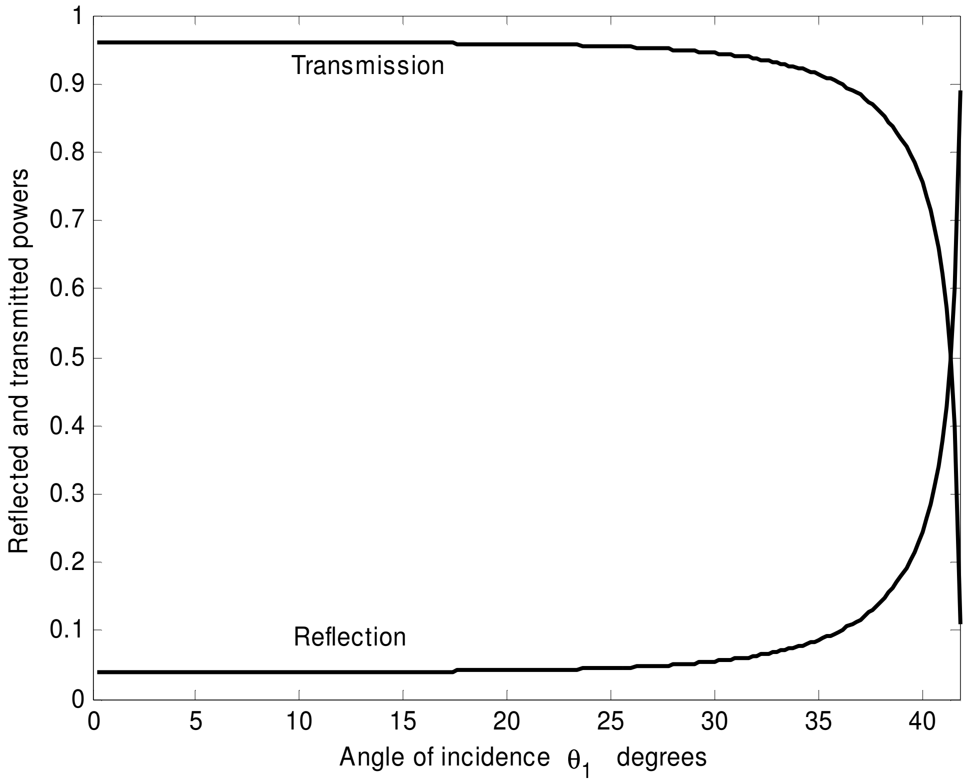
$$P : P_1 : P_2 = 1 : \left(\frac{E_1}{E}\right)^2 : n\left(\frac{E_2}{E}\right)^2 \frac{\cos\theta_2}{\cos\theta_1}$$

These are shown below for $n = 1.5$.



Recall that in these calculations, it has been assumed that the incident light is plane polarized at 45º to the parallel and senkrecht planes, so that the parallel and senkrecht amplitude components of the incident light are equal. Completely unpolarized incident light also has equal parallel and senkrecht amplitude components, so that the above graph also shows the reflection and transmission coefficients for unpolarized incident light. For n = 1.5, the reflection and transmission coefficients are equal for an angle of incidence of 82º.82. For any angle of incidence less than 60º, very much more light is transmitted than reflected., but, in the limit as $\theta_1 \rightarrow 90^{\circ}$, all the light is reflected.

We show below the reflection and transmission coefficients of internal reflection for angles of incidence from zero to the critical angle, which, for n = 1.5, is 41º.8. This is achieved merely by replacing 1.5 with 2/3 in the calculations.



Reflected and transmitted powers
Angle of incidence θ1 degrees
Transmission
Reflection
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
35
40