CHAPTER 16
CGS ELECTRICITY AND MAGNETISM

16.1 *Introduction*

We are accustomed to using MKS (metre-kilogram-second) units. A second, at one time defined as a fraction 1/86400 of a day, is now defined as 9 192 631 770 times the period of a hyperfine line emitted in the spectrum of the ^{133}Cs (caesium) atom. A metre was at one time defined as one ten-millionth of the length of a quadrant of Earth's surface measured from pole to equator. Later it was defined as the distance between two scratches on a platinum-iridium bar held on Paris. Still later, it was defined in terms of the wavelength of one or other of several spectral lines that have been used in the past for this purpose. At present, the metre is defined as the distance travelled by light *in vacuo* in a time of 1/(299 792 458) second. A kilogram is equal to the mass of a platinum-iridium cylinder held in Paris. The day may come when we are able to define a kilogram as the mass of so many electrons, but that day is not yet.

For electricity and magnetism, we extended the MKS system by adding an additional unit, the ampère, whose definition was given in Chapter 6, Section 6.2, to form the MKSA system. This in turn is a subset of SI (le Système International des Unités), which also includes the kelvin, the candela and the mole.

An older system of units, still used by some authors, was the CGS (centimetre-gram-second) system. In this system, a *dyne* is the *force* that will impart an acceleration of 1 cm s^{-2} to a mass of 1 gram. An *erg* is the *work* done when a force of one dyne moves its point of application through 1 cm in the line of action of the force. It will not take the reader a moment to see that a newton is equal to 10^5 dynes, and a joule is 10^7 ergs. As far as mechanical units are concerned, neither one system has any particular advantage over the other.

When it comes to electricity and magnetism, however, the situation is entirely different, and there is a huge difference between MKS and CGS. Part of the difficulty stems from the circumstance that electrostatics, magnetism and current electricity originally grew up as quite separate disciplines, each with its own system of units, and the connections between them were not appreciated or even discovered. It is not always realized that there are several version of CGS units used in electricity and magnetism, including hybrid systems, and countless conversion factors between one version and another. There are CGS *electrostatic* units (esu), to be used in electrostatics; CGS *electromagnetic* units (emu), to be used for describing magnetic quantities; and gaussian *mixed* units. In the gaussian mixed system, in equations that include both electrostatic quantities and magnetic quantities, the former were supposed to be expressed in esu and the latter in emu, and a conversion factor, given the symbol c, would appear in various parts of an equation to take account of the fact that some quantities were expressed in one system of units and others were expressed in another system. There was also the *practical* system of units, used in current electricity. In this, the ampère would be defined either in terms of the rate of electrolytic deposition of silver from a silver nitrate solution, or as exactly 0.1 CGS emu of current. The *ohm* would be defined in terms of the resistance of a column of mercury of defined dimensions, or again as exactly 10^9 emu of resistance. And a *volt* was 10^8 emu of potential difference. It will be seen already that, for every electrical quantity, several conversion factors between the different systems had to be known. Indeed, the MKSA system was devised specifically to avoid this proliferation of conversion factors.

Generally, the units in these CGS system have no particular names; one just talks about so many esu of charge, or so many emu of current. Some authors, however, give the names statcoulomb, statamp, statvolt, statohm ,etc., for the CGS esu of charge, current, potential difference and resistance, and abcoulomb, abamp, abvolt, abohm for the corresponding emu.

The difficulties by no means end there. For example, Coulomb's law is generally written as

$$F = \frac{Q_1 Q_2}{kr^2}. \tag{16.1.1}$$

It will immediately be evident from this that the permittivity defined by this equation differs by a factor of 4π from the permittivity that we are accustomed to. In the familiar equation generally used in conjunction with SI units, namely

$$F = \frac{Q_1 Q_2}{4\pi\epsilon r^2}, \tag{16.1.2}$$

the permittivity ϵ so defined is called the *rationalized* permittivity. The permittivity k of equation16.1.1 is the *unrationalized* permittivity. The two are related by $k = 4\pi\epsilon$. A difficulty with the unrationalized form is that a factor 4π appears in formulas describing uniform fields, and is absent from formulas describing situations with spherical symmetry.

Yet a further difficulty is that the magnitude of the CGS esu of charge is defined in such a way that the unrationalized free space permittivity has the numerical value 1 – and consequently it is normally left out of any equations in which it should appear. Thus equations as written often do not balance dimensionally, and one is deprived of dimensional analysis as a tool. Permittivity is regarded as a *dimensionless number*, and Coulomb's law for two charges *in vacuo* is written as

$$F = \frac{Q_1 Q_2}{r^2}. \tag{16.1.3}$$

The view is taken that electrical quantities can be expressed dimensionally in terms of mass, length and time only, and, from equation 16.1.3, it is asserted that the dimensions of electrical charge are

$$[Q] = \mathrm{M^{1/2} L^{3/2} T^{-1}}. \tag{16.1.4}$$

Because permittivity is regarded as a dimensionless quantity, the vectors **E** and **D** are regarded as dimensionally similar, and *in vacuo* they are *identical*. That is, *in vacuo*, there is no distinction between them.

When we come to CGS *electromagnetic units* all these difficulties reappear, except that, in the emu system, the free space *permeability* is regarded as a dimensionless number equal to 1, **B** and **H** are dimensionally similar, and *in vacuo* there is no distinction between them. The dimensions of electric charge in the CGS emu system are

$$[Q] = M^{1/2}L^{1/2}. \qquad 16.1.5$$

Thus the dimensions of charge are different in esu and in emu.

Two more highlights. The unit of capacitance in the CGS esu system is the centimeter, but in the CGS emu system, the centimetre is the unit of inductance.

Few users of CGS esu and emu fully understand the complexity of the system. Those who do so have long abandoned it for SI. CGS units are probably largely maintained by those who work with CGS units in a relatively narrow field and who therefore do not often have occasion to convert from one unit to another in this immensely complicated and physically unrealistic system.

Please don't blame *me* for this – I'm just the messenger!

In Sections 16.2, 16.3 and 16.4 I shall describe some of the features of the esu, emu and mixed systems. I shall not be giving a full and detailed exposition of CGS electricity, but I am just mentioning some of the highlights and difficulties. You are not going to like these sections very much, and will probably not make much sense of them. I suggest just skip through them quickly the first time, just to get some idea of what it's all about. *The practical difficulty that you are likely to come across in real life* is that you will come across equations and units written in CGS language, and you will want to know how to translate them into the SI language with which you are familiar. I hope to address that in Section 16.5, and to give you some way of translating a CGS formula into an SI formula that you can use and get the right answer from.

16.2 *The CGS Electrostatic System*

Definition. One CGS esu of charge (also known as the *statcoulomb*) is that charge which, if placed 1 cm from a similar charge *in vacuo*, will repel it with a force of 1 dyne.

The following exercises will be instructive.

Calculate, from the SI electricity that you already know, the force between two coulombs placed 1 cm from each other. From this, calculate how many CGS esu of charge there are in a coulomb. (I make it 1 coulomb = 2.998×10^9 esu. It will not escape your notice that this number is ten times the speed of light expressed in m s^{-1}.) Calculate the magnitude of the electronic charge in CGS esu. (I make it 4.8×10^{-10} esu. If, for example, you see that the potential energy of two electrons at a distance r apart is e^2/r, this is the number you must substitute for e. If you then express r in cm, the energy will be in ergs.)

Coulomb's law *in vacuo* is

$$F = \frac{Q_1 Q_2}{r^2}, \qquad 16.2.1$$

This differs from our accustomed formula (equation 16.1.2) in two ways. In the first place, we are using an *unrationalized* definition of permittivity, so that the familiar 4π is absent. Secondly, we are choosing units such that $4\pi\epsilon_0$ has the numerical value 1, and so we are omitting it from the equation.

While some readers (and myself!) will object, and say that equation 16.2.1 does not balance dimensionally, and is valid only if the quantities are expressed in particular units, others will happily say that equation 16.2.1 shows that the dimensions of Q are $\text{M}^{1/2}\text{L}^{3/1}\text{T}^{-1}$, and will not mind these extraordinary dimensions.

Electric field **E** is defined in the usual way, i.e. **F** = Q**E**, so that if the force on 1 esu of charge is 1 dyne, the field strength is 1 esu of electric field. (The esu of electric field has no name other than esu.) This is fine, but have you any idea how this is related to the SI unit of **E**, volt per metre? It requires a *great* deal of mental gymnastics to find out, so I'll just give the answer here, namely

$$1 \text{ CGS esu of } \mathbf{E} = 10^{-6}c \ \text{ V m}^{-1},$$

where

$$c = 2.997\,924\,58 \times 10^{10}.$$

Now the vector **D** is defined by **D** = k**E**, and since the permittivity is held to be a dimensionless number, **D** and **E** are held to be dimensionally similar ($\text{M}^{1/2}\text{L}^{-1/2}\text{T}^{-1}$ in fact). Further, since the free space permittivity is 1, *in vacuo* there is no distinction between **D** and **E**, and either can substitute for the other. So the conversion between CGS esu and SI for **D** is the same as for **E**? No! In SI, we recognize **D** and **E** as being physically different quantities, and **D** is expressed in coulombs per square metre. *Awful* mental gymnastics are needed to find the conversion, but I'll give the answer here:

$$1 \text{ CGS esu of } \mathbf{D} = \frac{10^5}{4\pi c} \ \text{ C m}^{-2}.$$

Once again, please don't blame me – I'm just the messenger! And be warned – it is going to get worse – much worse.

Potential Difference

If the work required to move a charge of 1 esu from one point to another is 1 erg, the potential difference between the points is 1 esu of potential difference, or 1 statvolt.

It is often said that an esu of potential difference is 300 volts, but this is just an approximation. The exact conversion is

$$1 \text{ statvolt} = 10^{-8} c \text{ V}.$$

Capacitance

If the potential difference across the plate of a capacitor is one statvolt when the capacitor holds a charge of one statcoulomb, the capacitance of the capacitor is one centimetre. (No – that's not a misprint.)

$$1 \text{ cm} = 10^9 c^{-2} \text{ F}.$$

Here is a sample of some formulas for use with CGS esu.

Potential at a distance r from a point charge Q *in vacuo* = Q/r.

Field at a distance r *in vacuo* from an infinite line charge of λ esu/cm = $2\lambda/r$.

Field *in vacuo* above an infinite charged plate bearing a surface charge density of σ esu/cm^2 = $2\pi\sigma$.

An electric dipole moment **p** is, as in SI, the maximum torque experienced by the dipole in unit electric field. A *debye* is 10^{-18} esu of dipole moment. The field at a distance r *in vacuo* along the axis of a dipole is $2p/r$.

Gauss's theorem: The total normal outward flux through a closed surface is 4π time the enclosed charge.

Capacitance of a plane parallel capacitor = $\dfrac{kA}{4\pi d}$.

Capacitance of an isolated sphere of radius a *in vacuo* = a. *Example*: What is the capacitance of a sphere of radius 1 cm? Answer: 1 cm. Easy, eh?

Energy per unit volume or an electric field = $E^2/(8\pi)$.

One more example before leaving esu. You will recall that, if a polarizable material is placed in an electrostatic field, the field **D** in the material is greater than $\varepsilon_0\mathbf{E}$ by the *polarization* **P** of the material. That is, $\mathbf{D} = \varepsilon\mathbf{E} + \mathbf{P}$. The equivalent formula for use with CGS esu is

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}.$$

And since $\mathbf{P} = \chi_e\mathbf{E}$ and $\mathbf{D} = k\mathbf{E}$, it follows that

$$k = 1 + 4\pi\chi_e.$$

At this stage you may want a conversion factor between esu and SI for all quantities. I'll supply one a little later, but I want to describe emu first, and then we can construct a table given conversions between all three systems.

16.3 *The CGS Electromagnetic System*

If you have been dismayed by the problems of CGS esu, you don't yet know what is in store for you with CGS emu. Wait for it:

Definition. One CGS emu of magnetic pole strength is that pole which, if placed 1 cm from a similar pole *in vacuo*, will repel it with a force of 1 dyne.

The system is based on the proposition that there exists a "pole" at each end of a magnet, and that point poles repel each other according to an inverse square law. Magnetic field strength **H** is defined as the force experienced by a unit pole situated in the field. Thus, if a pole of strength m emu is situated in a field of strength **H**, it will experience a force $\mathbf{F} = m\mathbf{H}$.

Definition. If a pole of strength 1 emu experiences a force of 1 dyne when it is situated in a magnetic field, the strength of the magnetic field is 1 *oersted* (Oe). It will probably be impossible for the reader at this stage to try to work out the conversion factor between Oe and A m^{-1}, but, for the record

$$1 \text{ Oe} = \frac{250}{\pi} \text{ A m}^{-1}.$$

Now hold on tight, for the definition of the unit of *electric current*.

Definition: One emu of current (1 *abamp*) is that steady current, which, flowing in the arc of a circle of length 1 cm and of radius 1 cm (i.e. subtending 1 radian at the centre of the circle) gives rise to a magnetic field of 1 oersted at the centre of the circle.

This will involve quite an effort of the imagination. First you have to imagine a current flowing in an arc of a circle. Then you have to imagine measuring the field at the centre of the circle by measuring the force on a unit magnetic pole that you place there.

It follows that, if a current I abamp flows in a circle of radius a cm, the field at the centre is of the circle is

$$H = \frac{2\pi I}{a} \text{ Oe}\cdot$$

The conversion between emu of current (abamp) and ampères is

$$1 \text{ emu} = 10 \text{ A}.$$

The Biot-Savart law becomes

$$dH = \frac{I\,ds\,\sin\theta}{r^2}.$$

The field at a distance *r in vacuo* from a long straight current *I* is

$$H = \frac{2I}{r}.$$

Ampère's law says that the line integral of **H** around a closed plane curve is 4π times the enclosed current. The field inside a long solenoid of *n* turns per centimetre is

$$H = 4\pi nI.$$

So far, no mention of **B**, but it is now time to introduce it. Let us imagine that we have a long solenoid of *n* turns per cm, carrying a current of *I* emu, so that the field inside it is $4\pi nI$ Oe. Suppose that the cross-sectional area of the solenoid is *A*. Let us wrap a single loop of wire tightly around the outside of the solenoid, and then change the current at a rate $\dot{I}$ so that the field changes at a rate $\dot{H} = 4\pi n\dot{I}$. An EMF will be set up in the outside (secondary) coil of magnitude $A\dot{H}$. If we now insert an iron core inside the solenoid and repeat the experiment, we find that the induced EMF is much larger. It is larger by a (supposed dimensionless) factor called the *permeability* of the iron. Although this factor is called the permeability and the symbols used is often μ, I am going to use the symbol κ for it. The induced EMF is now *A* times $\kappa\dot{H}$. We denote the product of μ and *H* with the symbol *B*, so that $B = \kappa H$. The magnitude of *B* inside the solenoid is

$$B = 4\pi\kappa nI.$$

It will be evident from the familiar SI version $B = \mu nI$ that the CGS emu definition of the permeability differs from the SI definition by a factor 4π. The CGS emu definition is called an *unrationalized* definition; the SI definition is *rationalized.* The relation between them is $\mu = 4\pi\kappa$.

In CGS emu, the permeability of free space has the value 1. Indeed the supposedly dimensionless unrationalized permeability is what, in SI parlance, would be the *relative permeability.*

The CGS unit of *B* is the *gauss* (G), and 1 G $= 10^{-4}$ T.

It is usually held that κ is a dimensionless number, so that *B* and *H* have the same dimensions, and, in free space, *B* and *H* are *identical.* They are identical not only numerically, but there is physically no distinction between them. Because of this, the unit *oersted* is rarely heard, and it is common to hear the unit *gauss* used haphazardly to describe either *B* or *H*.

The scalar product of **B** and area is the magnetic flux, and its CGS unit, G cm^2, bears the name the *maxwell.* The rate of change of flux in maxwells per second will give you the induced EMF in emus (abvolts). An abvolt is 10^{-8} V.

The subject of *magnetic moment* has caused so much confusion in the literature that I shall devote an entire future chapter to it rather than try to do it here.

I end this section by giving the CGS emu version of *magnetization.* The familiar $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$ becomes, in its CGS emu guise, $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$. The magnetic susceptibility χ_m is defined by $\mathbf{M} = \chi_m\mathbf{H}$. Together with $\mathbf{B} = \kappa\mathbf{H}$, this results in $\kappa = 1 + 4\pi\chi_m$.

16.4 *The Gaussian Mixed System*

A problem arises if we are dealing with a situation in which there are both "electrostatic" and "electromagnetic" quantities. The "mixed system", which is used *very frequently*, in CGS literature, uses esu for quantities that are held to be "electrostatic" and emu for quantities that are held to be "electromagnetic", and it seems to be up to each author to decide which quantities are to be regarded as "electrostatic" and which are "electromagnetic. Because different quantities are to be expressed in different sets of units within a single equation, the equation must include the conversion factor $c = 2.997\,924\,58 \times 10^{10}$ in strategic positions within the equation.

The most familiar example of this is the equation for the force **F** experienced by a charge Q when it is moving with velocity **v** in an electric field **E** and a magnetic field **B**. This equation is liable to appear either as

$$F = Q\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{H}}{c}\right) \qquad 16.4.1$$

or as

$$F = Q\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right). \qquad 16.4.2$$

It can appear in either of these forms because, if CGS emu are used, B and H are numerically equal *in vacuo*. The conversion factor c appears in these equations, because it is understood (by those who understand CGS units) that Q and E are to be expressed in esu, while B or H is to be expressed in emu, and the conversion factor c is necessary to convert it to esu.

It should be noted that in all previous chapters in these notes, equations balance dimensionally, and the equations are valid *in any coherent system of units, not merely SI.* Difficulties arise, of course, if you write an equation that is valid only so long as a particular set of units is used, and even more difficulties arise if some quantities are to be expressed in one system of units, and other quantities are to be expressed in another system of units.

An analogous situation is to be found in some of the older books on thermodynamics, where it is possible to find the following equation:

$$C_P - C_V = R/J. \qquad 16.4.3$$

This equation expresses the difference in the specific heat capacities of an ideal gas, measured at constant pressure and at constant volume. In equation 16.4.3, it is understood that C_P and C_V are to be expressed in calories per gram per degree, while the universal gas constant is to be expressed in ergs per gram per degree. The factor J is a conversion factor between erg and calories. Of course the sensible way to write the equation is merely

$$C_P - C_V = R. \qquad 16.4.4$$

This is valid *whatever* units are used, be they calories, ergs, joules, British Thermal Units or kWh, as long as all quantities are expressed in the same units. Yet it is truly extraordinary how many electrical equations are to be found in the literature, in which different units are to be used for dimensionally similar quantities.

Maxwell's equations may appear in several forms. I take one at random from a text written in CGS:

$$\mathrm{div}\mathbf{B} = 0, \qquad 16.4.5$$

$$\mathrm{div}\mathbf{D} = 4\pi\rho, \qquad 16.4.6$$

$$c\,\mathbf{curlH} = \dot{\mathbf{D}} + 4\pi\mathbf{J}, \qquad 16.4.7$$

$$c\,\mathbf{curlE} = -\dot{\mathbf{B}}. \qquad 16.4.8$$

The factor c occurs as a conversion factor, since some quantities are to be expressed in esu and some in emu. The 4π arises because of a different definition (unrationalized) of permeability. In some versions there may be no distinction between **B** and **H**, or between **E** and **D**, and the 4π and the c may appear in various places in the equations.

(It may also be remarked that, in the earlier papers, and in Maxwell's original writings, vector notation is not used, and the equations appear extremely cumbersome and all but incomprehensible to modern eyes.)

16.5 *Conversion Factors*

By this time, you are completely bewildered, and you want nothing to do with such a system. Indeed you may even be wondering if I made it all up, so irrational does it appear to be. You would like to ignore it all completely. But you cannot ignore it, because, in your reading, you keep coming across formulas that you need, but you don't know what units to use, or whether there should be a 4π in the formula, or whether there is a permittivity or permeability missing from the equation because the author happens to be using some set of units in which one or the other of these quantities has the numerical value 1, or whether the H in the equation should really be a B, or the E a D.

Is there anything I can do to help?

What I am going to do in this section is to list a number of conversion factors between the different systems of units. This may help a little, but it won't by any means completely solve the problem. Really to try and sort out what a CGS equation means requires some dimensional analysis, and I shall address that in section 16.6

In the conversion factors that I list in this section, the symbol c stands for the number 2.997 924 58 $\times$ 10^{10}, which is numerically equal to the speed of light expressed in cm s^{-1}. The abbreviation "esu" will mean CGS electrostatic unit, and "emu" will mean CGS electromagnetic unit. A prefix "stat" to a unit implies that it is an esu; a prefix "ab" implies that it is an emu. I list the conversion factors for each quantity in the form "1 SI unit = so many esu = so many emu".

I might mention that people will say that "SI is full of conversion factors". The fact is that SI is a unified coherent set of units, and it has *no* conversion factors. Conversion factors are characteristic of CGS electricity and magnetism.

Quantity of Electricity (Electric Charge)

1 coulomb = $10^{-1}c$ statcoulomb = 10^{-1} emu

Electric Current

1 amp = $10^{-1}c$ esu = 10^{-1} abamp

Potential Difference

1 volt = $10^8/c$ statvolt = 10^8 emu

Resistance

1 ohm = $10^9/c^2$ esu = 10^9 abohm

Capacitance

1 farad = $10^{-9}c^2$ esu = 10^{-9} emu

Inductance

1 henry = $10^9/c^2$ esu = 10^9 emu

Electric Field E

$$1 \text{ V m}^{-1} = 10^6/c \text{ esu} = 10^6 \text{ esu}$$

Electric Field D

$$1 \text{ C m}^{-2} = 4\pi \times 10^{-5} c \text{ esu} = 4\pi \times 10^{-5} \text{ emu}$$

Magnetic Field B

$$1 \text{ tesla} = 10^4/c \text{ esu} = 10^4 \text{ gauss}$$

Magnetic Field H

$$1 \text{ A m}^{-1} = 4\pi \times 10^{-3} c \text{ esu} = 4\pi \times 10^{-3} \text{ oersted}$$

Magnetic B-flux Φ_B

$$1 \text{ weber} = 10^8/c \text{ esu} = 10^8 \text{ maxwell}$$

16.6 *Dimensions*

A book says that the equivalent width W, in wavelength units, of a spectrum line, is related to the number of atoms per unit area in the line of sight, N, by

$$W = \frac{\pi e^2 N \lambda^2}{mc^2}. \qquad 16.6.1$$

Is this formula all right in *any* system of units? Can I use SI units on the right hand side, and get the answer in metres? Or must I use a particular set of units in order to get the right answer? And if so, which units?

A book says that the rate at which energy is radiated, P, from an accelerating charge is

$$P = \frac{2e^2 \ddot{x}^2}{c^3}. \qquad 16.6.2$$

Is this correct? Is c the speed of light, or is it merely a conversion factor between different units? Or is one of the cs a conversion factor, and the other two are the speed of light?

It *is* possible to find the answer to such bewildering questions, if we do a bit of dimensional analysis. So, before trying to answer these specific questions (which I shall do later as examples) I am going to present a table of dimensions. I already gave a table of dimensions of electrical quantities in Chapter 11, in terms of M, L, T and Q, but that table won't be particularly helpful in the present context.

I pointed out in Section 16.1 of the present chapter that Coulomb's law is often written in the form

$$F = \frac{Q_1 Q_2}{r^2}. \qquad 16.6.3$$

Consequently the dimensions of Q are held to be $[Q] = \mathrm{M}^{1/2}\mathrm{L}^{3/2}\mathrm{T}^{-1}$. But we know that a permittivity is missing from the denominator of equation 16.6.3, because the writer intends his formula to be restricted to a particular set of units such that k or $4\pi\epsilon_0 = 1$. In order to detect whether a permittivity has been omitted from an equation, we need a table in which the dimensions of electrical quantities are given not in terms of M, L, T and Q as in Chapter 11, but in terms of M, L, T and ϵ, and this is what I am just about to do. However, often it is the *permeability* that has been omitted from an equation, and, in order to detect whether this is so, I am also supplying a table in which the dimensions of electrical quantities are given in terms of M, L, T and μ.

If, from dimensional analysis, you find that an expression is dimensionally wrong by a power of the permittivity, insert $4\pi\epsilon_0$ in the appropriate part of the equation. If you find that an expression is dimensionally wrong by a power of the permeability, insert $\mu_0/(4\pi)$ in the appropriate part of the equation. If you find that the equation is wrong by LT^{-1}, insert or delete c as appropriate. Your equation will then balance dimensionally and will be ready for use in *any* coherent system of units, including SI. This procedure will probably work in most cases, but I cannot guarantee that it will work in all cases, because it cannot deal with those (frequent!) cases in which the formula given is plain wrong, whatever units are used!

	M	L	T	ε		M	L	T	μ
Electric charge	$\frac{1}{2}$	$\frac{3}{2}$	-1	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$
Electric dipole moment	$\frac{1}{2}$	$\frac{5}{2}$	-1	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{3}{2}$	0	$-\frac{1}{2}$
Current	$\frac{1}{2}$	$\frac{3}{2}$	-2	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	-1	$-\frac{1}{2}$
Potential difference	$\frac{1}{2}$	$\frac{1}{2}$	-1	$-\frac{1}{2}$		$\frac{1}{2}$	$\frac{3}{2}$	-2	$\frac{1}{2}$
Resistance	0	-1	1	-1		0	1	-1	1
Resistivity	0	0	1	-1		0	2	-1	1
Conductance	0	1	-1	1		0	-1	1	-1
Conductivity	0	0	-1	1		0	-2	1	-1
Capacitance	0	1	0	1		0	-1	2	-1
Electric field *E*	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	-2	$\frac{1}{2}$
Electric field *D*	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$		$\frac{1}{2}$	$-\frac{3}{2}$	0	$-\frac{1}{2}$
Electric flux Φ_E	$\frac{1}{2}$	$\frac{3}{2}$	-1	$-\frac{1}{2}$		$\frac{1}{2}$	$-\frac{5}{2}$	-2	$\frac{1}{2}$
Electric flux Φ_D	$\frac{1}{2}$	$\frac{3}{2}$	-1	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$
Permittivity	0	0	0	1		0	-2	2	-1
Magnetic field *B*	$\frac{1}{2}$	$-\frac{3}{2}$	0	$-\frac{1}{2}$		$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$
Magnetic field *H*	$\frac{1}{2}$	$\frac{1}{2}$	-2	$\frac{1}{2}$		$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$
Magnetic flux Φ_B	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$
Magnetic flux Φ_H	$\frac{1}{2}$	$\frac{5}{2}$	-2	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	-1	$-\frac{1}{2}$
Permeability	0	-2	2	-1		0	0	0	1
Magnetic vector potential	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$
Inductance	0	-1	2	-1		0	1	0	1

Now let's look at the equation for equivalent width of a spectrum line:

$$W = \frac{\pi e^2 N \lambda^2}{mc^2}. \qquad 16.6.1$$

Here $[W] = \mathrm{L}$ and $[N] = \mathrm{L}^{-2}$. By making use of the table we find that the dimensions of the right hand side are Lε. There is therefore a $4\pi\epsilon_0$ missing from the denominator, and the equation should be

$$W = \frac{\pi e^2 N \lambda^2}{4\pi\epsilon_0 mc^2}. \qquad 16.6.4$$

How about the rate at which energy is radiated from an accelerating charge?

$$P = \frac{2e^2\ddot{x}^2}{c^3}. \qquad 16.6.2$$

Power has dimensions ML^2T^{-3}, whereas the dimensions of the right hand side are $ML^2T^{-3}\varepsilon$, so again there is a $4\pi\varepsilon_0$ missing from the denominator and the formula should be

$$P = \frac{2e^2\ddot{x}^2}{4\pi\varepsilon_0 c^3}. \qquad 16.6.5$$

It is often the case that there is a $4\pi\varepsilon_0$ missing from the denominator is formulas that have an e^2 upstairs.

"Electromagnetic" formulas often give more difficulty. For example, one book says that the energy per unit volume in a magnetic field *in vacuo* is $\frac{B^2}{8\pi}$, while another says that is it is $\frac{H^2}{8\pi}\cdot$ Which is it (if indeed it is either)? Energy per unit volume has dimensions $ML^{-1}T^{-2}$. The dimensions of B^2 are $ML^{-1}T^{-2}\mu$. The equation given is therefore wrong dimensionally by permeability, and the equation should be divided by $\mu_0/(4\pi)$ to give $B^2/(2\mu_0)$, which is correct. On the other hand, the dimensions of H^2 are $ML^{-1}T^{-2}\mu^{-1}$, so perhaps we should *multiply* by $\mu_0/(4\pi)$? But this does not give a correct answer, and it exemplifies some of the many difficulties that are caused by writing formulas that do not balance dimensionally and are intended to be used only with a particular set of units. The situation is particularly difficult with respect to *magnetic moment*, a subject to which I shall devote the next chapter.